

**The G_2 -MSSM -
An M Theory motivated model of Particle Physics**

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We continue our study of the low energy implications of M theory vacua on G_2 manifolds, undertaken in [1, 2], where it was shown that the moduli can be stabilized and a TeV scale generated, with the Planck scale as the only dimensionful input. A well-motivated phenomenological model - the G_2 -MSSM, can be naturally defined within the above framework. In this paper, we study some of the important phenomenological features of the G_2 -MSSM. In particular, the soft supersymmetry breaking parameters and the superpartner spectrum are computed. The G_2 -MSSM generically gives rise to light gauginos and heavy scalars with wino LSPs when one tunes the cosmological constant. Electroweak symmetry breaking is present but fine-tuned. The G_2 -MSSM is also naturally consistent with precision gauge coupling unification. The phenomenological consequences for cosmology and collider physics of the G_2 -MSSM will be reported in more detail soon.

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I. INTRODUCTION AND SUMMARY

This year we will enter the LHC era. The LHC experiments will likely dominate particle physics for many years. String/ M theory phenomenology has to address many challenges, the most important of which are related to dynamical issues, such as supersymmetry breaking, moduli stabilization and explaining the Hierarchy between the Electroweak and Planck scales, etc. Successfully addressing these opens the possibility to construct models of particle physics beyond the Standard-Model (SM) within the framework of string/ M theory and study them to the extent that testable predictions for real observables at the LHC (as well as other branches of particle physics and cosmology) can be made. Different approaches will in general give rise to different patterns of signatures; this could in principle distinguish among different possibilities [3, 4]. In carrying out this program in string/ M theory, it has to be kept in mind that the properties of beyond-the-Standard-Model (BSM) particle physics models are intimately connected to the dynamical issues mentioned above. This is because the masses and couplings of the particle physics models depend on the properties of the vacuum (or class of vacua) of the underlying string/ M theory constructions, including the values of the moduli in the vacuum.

Substantial progress has been made in the past few years towards addressing the dynamical issues of moduli stabilization, supersymmetry breaking and explaining the Hierarchy, within various corners of the entire M theory moduli space in a reliable manner. The most popular one is the Type IIB corner [5]. There have also been explicit semi-realistic constructions of the visible sector within the Type IIB setup. While there still do not exist explicit realizations incorporating mechanisms for solving all the above mentioned dynamical problems as well as incorporating a realistic visible sector within one particular construction, it is possible to consider *frameworks* in which the relevant effects of the underlying mechanisms may reasonably be assumed to exist in a self-consistent manner. For example, within Type IIB string theory, one popular *framework* is warped flux compactification of Type IIB string theory to four dimensions in the presence of $D3$, anti $D3$ and $D7$ -branes. The closed string fluxes stabilize the dilaton and complex structure moduli whilst non-perturbative effects stabilize the Kähler moduli. The anti- $D3$ -branes reside at the end of the warped throat and break supersymmetry while the visible sector (composed of $D3/D7$ -branes) resides in the bulk of the internal manifold. Supersymmetry breaking at the end of the throat is mediated to the visible sector by the (higher dimensional) gravity multiplet, i.e. the moduli. In this *framework*, also known as “mirage mediation”, assuming the existence of all the relevant effects of the underlying microscopic string compactification, one can incorporate them in a self-consistent manner and then study the phenomenological consequences for low energy observables. Such studies have been carried out in the literature [6].

The object of interest in this paper is a *framework* arising from another weakly coupled limit: the low energy limit of M theory. It was shown in [1, 2] that, with reasonable assumptions about microscopic structure of the underlying construction, $\mathcal{N}=1$ fluxless compactifications of M theory on G_2 manifolds can generate the hierarchy between the electroweak and Planck scales and stabilize all the moduli in a dS vacuum. This framework offers the possibility for studying the consequences for low energy phenomenology. That is the goal of this paper. In a series of companion papers, more detailed applications for cosmology and collider physics will be presented.

The results obtained from the analysis of vacua within the above framework are quite interesting and lead us to define a class of particle physics models which we call the G_2 -MSSM. Many detailed properties

of these models are derived from M theory, though we also add some necessary assumptions due to theoretical uncertainties about the M theory framework.

The G_2 -MSSM models have a distinctive spectrum. One finds that, at the compactification scale ($\sim M_{\text{unif}}$), the gauginos are light ($\lesssim 1$ TeV) and are suppressed compared to the trilinears, scalar and Higgsino masses which are roughly equal to the gravitino mass ($\sim 30 - 100$ TeV). At the electroweak scale, the lightest top squark turns out to be significantly lighter than the other squarks ($\sim 1 - 10$ TeV) because of RGE running. In addition, there are significant finite threshold corrections to bino and wino masses from the large Higgsino mass. Radiative electroweak symmetry breaking is generic and $\tan\beta$ is naturally predicted from the structure of the high scale theory to be of $\mathcal{O}(1)$. (Theoretical predictions of $\tan\beta$ are fairly rare). The value of m_Z is fine-tuned, however, implying the existence of the Little-hierarchy problem, which, because of the larger scalar masses is worse than the usual little hierarchy.

The class of vacua within this framework also tend to be consistent with precision gauge unification [7]. The LSP usually turns out to be wino for solutions consistent with precision gauge unification. Though the thermal relic abundance of wino LSPs is quite low, an analysis of the cosmological evolution of the moduli shows that non-thermal production of wino LSPs provides about the correct amount of dark-matter¹. The collider phenomenology of the vacua within this framework is also quite distinctive, giving rise to many b -jets from multi-top production and short charged track stubs from the decay of the wino-like chargino.

The paper is organized as follows. In the rest of this section, the results obtained in [1, 2] are summarized and the assumptions about the microscopic structure needed to define the framework and make contact with low energy physics are specified. In particular, G_2 -MSSM vacua, whose detailed phenomenology will be studied, are motivated and introduced. Readers interested only in phenomenological results may skip section I. In section II, the computation of soft supersymmetry breaking parameters at the unification scale consistent with all relevant constraints is presented. Section III deals with RG evolution, calculation of the superpartner spectrum and Electroweak Symmetry Breaking (EWSB). Section IV studies precision gauge coupling unification and its relation to gaugino masses. In section V, we analyze the nature of the LSP and its connection to the Dark Matter (DM) relic density and the cosmological moduli problem. A brief discussion about benchmark spectra and collider phenomenology of the class of vacua within this framework appears in section VI. We conclude in section VII. This is followed by an Appendix discussing some technical details about the computation of the quantity P_{eff} that enters in tuning the cosmological constant and in the gaugino mass suppression, constraints on “microscopic” parameters, and threshold corrections to gaugino masses from heavy states.

A. Summary of Results about G_2 Vacua

In order to make our discussion of G_2 phenomenology self-contained, it is helpful to summarize the essential results for the fluxless M theory de Sitter vacua described in [1, 2] and explain our conventions and notation. Readers interested only in phenomenological results may skip this section. M theory compactifications on singular G_2 manifolds are interesting in the sense that they give rise to $\mathcal{N} = 1$

¹ Preliminary.

supersymmetry in four dimensions with non-Abelian gauge groups and chiral fermions. The non-Abelian gauge fields are localized along three-dimensional submanifolds of the seven extra dimensions whereas chiral fermions are supported at points at which there is a conical singularity [8, 9, 10, 11]. As explained in the introduction, in order to look at phenomenological consequences of these compactifications in a reliable manner, one has to address the dynamical issues of moduli stabilization, supersymmetry breaking and generation of the Hierarchy.

As explained in [1, 2], one is interested in the zero flux sector since then the moduli superpotential is entirely non-perturbative. This is crucial for both stabilizing the moduli and generating the Hierarchy naturally as we will review. Fluxes generate a large superpotential and, unless there is a mechanism to obtain an exponentially large volume of the extra dimensions, G_2 compactifications with flux will not generate a small mass scale, such as the TeV scale.

We assume that the G_2 manifolds which we consider have singularities giving rise to two non-Abelian, asymptotically free gauge groups. This implies that they undergo strong gauge dynamics at lower energies leading to the generation of a non-perturbative superpotential. At least one of the hidden sectors is assumed to contain light charged matter fields Q and \tilde{Q} (with $N_f < N_c$) as well. There could be other matter fields which are much heavier and decouple well above the corresponding strong coupling scale. Thus in the minimal² framework, one has two hidden sectors living on three-manifolds with gauge group $G_a \times G_b$ undergoing strong gauge dynamics, one of them having a pair of massless charged matter fields transforming in the (anti)fundamental representation of the gauge group. Of course, in addition, it is assumed that there is another three-manifold on which the observable sector gauge theory with the appropriate chiral matter content lives. This will be discussed more in the next subsection. This set of assumptions about the compactification manifold gives a working definition of the *framework*.

The $\mathcal{N} = 1$ supergravity theory obtained in four dimensions is then characterized by the following hidden sector superpotential:

$$W = m_p^3 \left(C_1 P \phi^{-(2/P)} e^{ib_1 f_1} + C_2 Q e^{ib_2 f_2} \right); \quad b_1 = \frac{2\pi}{P}, \quad b_2 = \frac{2\pi}{Q} \quad (1)$$

Here $\phi \equiv \det(Q\tilde{Q})^{1/2} = (2Q\tilde{Q})^{1/2}$ is the effective meson field (for one pair of massless quarks) and P and Q are proportional to one loop beta function coefficients of the two gauge groups which are completely determined by the gauge group and matter representations. For concreteness we can consider the gauge group to be $SU(Q) \times SU(P+1)$ with one vector like family of quarks charged under $SU(P+1)$. The normalization constants C_1 and C_2 are calculable, given a particular G_2 -manifold. $f_{1,2}$ are the (tree-level) gauge kinetic functions of the two hidden sectors which in general are different from each other. Schematically, the superpotential of each hidden sector is just equal to the strong coupling scale of the corresponding gauge theory, i.e. $W \sim \Lambda_1^3 + \Lambda_2^3$. The vacuum structure of the supergravity theory with this superpotential is quite rich, but in general can only be studied numerically. A special case exists however, when it is possible to study the vacua semi-analytically. This is when the two three-manifolds on which the hidden sector gauge fields are localised are in the same homology class, which in terms of

² More complicated situations are possible, some of them are discussed in [1, 2].

gauge kinetic function then implies:

$$f_1 = f_2 \equiv f_{\text{hid}} = \sum_{i=1}^N N_i z_i; \quad z_i = t_i + i s_i. \quad (2)$$

In the above equation, s_i are the N geometric moduli of the G_2 manifold (intuitively, these characterise the sizes of the 3-cycles in the G_2 manifold), while t_i are the axionic components coming from the 3-form field C_{IJK} of eleven-dimensional supergravity. The N_i are integers which are determined by the homology class of the hidden sector 3-cycles.

The supergravity potential is fully specified once the Kähler potential is given. The Kähler potential for matter fields in general is hard to compute from first principles. However, owing to the fact that matter fields are localized at points inside the seven dimensional manifold V_7 , it is reasonable to assume that the matter Kähler potential is approximately canonical at leading order. Then, the Kähler potential is given by:

$$K/m_p^2 = -3 \ln(4\pi^{1/3} V_7) + \bar{\phi} \phi \quad (3)$$

where $V_7 \equiv \frac{\text{Vol}(X)}{l_{11}^7}$ is the volume of the G_2 manifold X in units of the eleven-dimensional Planck length l_{11} , and is a homogenous function of the s_i of degree $7/3$. A simple and reasonable ansatz therefore is $V_7 = \prod_{i=1}^N s_i^{a_i}$ with a_i positive rational numbers subject to the constraint $\sum_{i=1}^N a_i = \frac{7}{3}$ [12]. Many qualitative results about moduli stabilization do not seem to rely on this special form of V_7 , but this form of V_7 is useful since it gives an $N - 1$ parameter family of Kähler potentials consistent with G_2 -holonomy, which are tractable. In a basis in which the Kähler potential is given by (3), the gauge kinetic function is generically a function of all the moduli, i.e. $N_i \neq 0, i = 1, 2, \dots, N$.

In general the scalar potential of the supergravity theory determined by W and K is a reasonably complicated function of all the moduli. Therefore, one expects to find isolated meta-stable minima, which indeed turns out to be the case, as explained in detail in [1]. The values of the moduli at the minima are completely determined by the *microscopic constants*³ - $\{a_i, N_i, C_1, C_2, P, Q, N; i = 1, 2, \dots, N\}$ which characterize the framework. Given a particular G_2 -manifold consistent with our assumptions, all of these constants are calculable in principle. Therefore, given a *particular* G_2 -manifold within the framework, one obtains a *particular* set of microscopic constants and a *particular* 4d $\mathcal{N} = 1$ supergravity theory.

To find the minima of the moduli potential V explicitly, one first stabilizes the axionic components of the complex moduli and the phase of ϕ . Then one minimizes the potential with respect to s_i and $|\phi|$, which leads to $N + 1$ equations $\partial_{s_i} V = 0$ and $\partial_{|\phi|} V = 0$ (for N moduli). To solve these equations analytically, we consider the class of solutions in which the volume of the hidden sector three-manifold $V_{\hat{Q}}$ supporting the hidden sector gauge groups is large. This allows us to reduce the first set of N equations into just two simple equations, which can be solved order by order in a $1/V_{\hat{Q}}$ expansion. Physically, this expansion can be understood as an expansion in terms of the small gauge coupling of the hidden sector - $(\alpha_0)_{\text{hid}}$ which is self-consistent since our hidden sectors are assumed to be asymptotically free. The

³ These are called "microscopic" because they determine the effective lagrangian at the compactification ($\sim M_{\text{unif}}$) scale.

solution corresponding to a metastable minimum with spontaneously broken supersymmetry is given by

$$s_i = \frac{a_i}{N_i} \frac{3}{14\pi} \frac{P_{\text{eff}} Q}{Q - P} + \mathcal{O}(P_{\text{eff}}^{-1}), \quad (4)$$

$$|\phi|^2 = 1 - \frac{2}{Q - P} + \sqrt{1 - \frac{2}{Q - P}} + \mathcal{O}(P_{\text{eff}}^{-1}), \quad (5)$$

where $P_{\text{eff}} \equiv P \ln(C_1/C_2)$. The natural values of P and Q are expected to lie between $\mathcal{O}(1)$ and $\mathcal{O}(10)$. It is easy to see that a large P_{eff} corresponds to small α for the hidden sector

$$(\alpha_0^{-1})_{\text{hid}} = \text{Im}(f_{\text{hid}}) \approx \frac{Q}{2\pi(Q - P)} P_{\text{eff}} \quad (6)$$

implying that the expansion is effectively in P_{eff}^{-1} . The ϕ dependence of the potential at the minimum is essentially

$$V_0 \sim m_{3/2}^2 M_P^2 \left[|\phi|^4 + \left(\frac{4}{Q - P} + \frac{14}{P_{\text{eff}}} - 3 \right) |\phi|^2 + \left(\frac{2}{Q - P} + \frac{7}{P_{\text{eff}}} \right) \right] \quad (7)$$

Therefore, the vacuum energy vanishes if the discriminant of the above expression vanishes, i.e. if

$$P_{\text{eff}} = \frac{28(Q - P)}{3(Q - P) - 8}. \quad (8)$$

The above condition is satisfied when the contribution from the F -term of the meson field (F_ϕ) to the scalar potential cancels that from the $-3m_{3/2}^2$ term. In this vacuum, the F -term of the moduli F_i are much smaller than F_ϕ . In fact, all the F_i vanish at the leading order in the $1/V_{\hat{Q}}$ expansion. For $Q - P \leq 2$, there is no solution as either s_i or $|\phi|^2$ become negative. The first non-trivial solution occurs when $Q - P = 3$ for which $P_{\text{eff}} = 84$ is required to get a vanishing vacuum energy (to leading order). The appearance of the integer 84 is closely related to the dimensionality of the G_2 manifold (which is 7) and that of the three-manifold (which is 3). Other choices of $Q - P$ are also possible theoretically, but are not interesting because of the following reasons: a) the corresponding solutions, if they are to remain in the supergravity regime, require the G_2 manifold to have a rather small number of moduli N , since $N < \frac{14Q}{(3(Q-P)-8)\pi} [1]$. It is unlikely that G_2 -manifolds with such few moduli are capable of containing the $MSSM$ spectrum, which has more than a hundred relevant couplings. b) these solutions generically lead to an extremely high susy breaking scale as will be seen in section II A on the gravitino mass. So phenomenologically interesting G_2 compactifications arise only for the case $Q - P = 3$ and $P_{\text{eff}} = 84$.

Some comments on the requirement of $P_{\text{eff}} = 84$ are in order. First of all, it is only a leading order result for the potential in $(\alpha_0^{-1})_{\text{hid}}$ expansion. In fact, including higher order $(\alpha_0^{-1})_{\text{hid}}$ corrections leads to the requirement $P_{\text{eff}} \approx 83$ [1]. The potential will also receive higher order corrections in the M theory expansion which will change the requirement for P_{eff} (probably by a small amount). One important good feature of the framework is that these higher order corrections to the vacuum energy have little effect on phenomenologically relevant quantities. Therefore, it is sufficient to tune the vacuum energy to leading order as long as one is interested in phenomenological consequences. From a microscopic point of view, however, there are two issues - a) Is it possible to realize a large value of P_{eff} from explicit constructions? and b) Can the values of P_{eff} scan finely enough such that one can obtain the observed tiny value of the cosmological constant? Regarding a), one notices that P_{eff} depends on the detailed structure of the

hidden sector and is completely model dependent. For particular realizations of the hidden sector, P_{eff} can be computed. A detailed discussion about P_{eff} is given in Appendix A. However computing P_{eff} in more general cases is difficult because of our limited knowledge of possible three-dimensional submanifolds of G_2 manifolds. In our analysis, we have assumed that three-manifolds exist for which it is possible to obtain a large P_{eff} . The situation regarding b) is even less known. This is because very little is known in general about the set of all compact G_2 manifolds. Duality arguments do suggest that the set of compact G_2 -manifolds is larger than the space of compact Calabi-Yau threefolds. Unfortunately, there do not currently exist any concrete ideas about that space either! In our work, we have assumed effectively that the space of G_2 manifolds scans P_{eff} finely enough such that vacua exist with values of the cosmological constant as observed.

B. The Observable Sector - Introducing the G_2 -MSSM

In these compactifications, as mentioned earlier, the observable sector gauge theory resides on a three-manifold different from the one supporting the hidden sector. The observable sector three-manifold is assumed to contain conical singularities at which chiral matter is supported. Since two three-manifolds in a seven dimensional manifold generically do not intersect each other, this implies that the supersymmetry breaking generated by strong gauge dynamics in the hidden sector is generically mediated to the visible sector by the (higher dimensional) gravity multiplet. This gives rise to gravity (moduli) mediation. However, as will be seen later, anomaly mediated contributions will also play an important role for the gaugino masses.

In our analysis henceforth, we will assume a GUT gauge group in the visible sector which is broken to the SM gauge group, with at least an MSSM chiral spectrum, by background gauge fields (Wilson lines). This assumption is well motivated by considering the duality to the $E_8 \times E_8$ heterotic string on a Calabi-Yau threefold. For simplicity, we will present our results for the $SU(5)$ GUT group breaking to the SM group and just an MSSM chiral spectrum, but all our results should hold for other GUT groups breaking in the same way as well.

To summarize, the full low energy $\mathcal{N} = 1$ Supergravity theory of the visible and hidden sectors at the compactification scale ($\sim M_{\text{unif}}$) is defined by the following:

$$\begin{aligned}
K/m_p^2 &= \left(-3 \ln(4\pi^{1/3} V_7) + \bar{\phi}\phi \right) + \tilde{K}_{\bar{\alpha}\beta}(s_i) \bar{\Phi}_{MSSM}^{\bar{\alpha}} \Phi_{MSSM}^{\beta} + (Z(s_i) H_u H_d + h.c.) + \dots \\
W &= m_p^3 \left(C_1 P \phi^{-(2/P)} e^{ib_1 f_1} + C_2 Q e^{ib_2 f_2} \right) + Y'_{\alpha\beta\gamma} \Phi_{MSSM}^{\alpha} \Phi_{MSSM}^{\beta} \Phi_{MSSM}^{\gamma} \\
f_1 &= f_2 \equiv f_{\text{hid}} = \sum_i N^i z_i; \quad \text{Im}(f_{vis}^0) = \sum_i N_{vis}^i s_i \equiv V_{\hat{Q}_{vis}}
\end{aligned} \tag{9}$$

The visible sector is thus characterized by the Kähler metric $\tilde{K}_{\bar{\alpha}\beta}$ and un-normalized Yukawa couplings $Y'_{\alpha\beta\gamma}$ of the visible sector chiral matter fields Φ_{MSSM}^{α} and the (tree-level) gauge kinetic function f_{vis}^0 of the visible sector gauge fields. In addition, as is generically expected in gravity mediation, a non-zero coefficient Z of the Higgs bilinear is assumed. In general there can also be a mass term (μ') in the superpotential W , but as explained in [13], natural discrete symmetries can exist which forbid it, in order to solve the doublet-triplet splitting problem. However, the Giudice-Masiero mechanism in general generates effective μ and $B\mu$ parameters of $\mathcal{O}(m_{3/2})$.

The Kähler metric $\tilde{K}_{\bar{\alpha}\beta}$ will be discussed in section II B. The visible sector gauge couplings are determined by the gauge kinetic function f_{vis} which is an integer linear combination of the moduli with the integers determined by the homology class of the three-manifold \hat{Q} which supports the visible gauge group. Because of a GUT-like spectrum, the MSSM gauge couplings are unified at M_{unif} giving rise to the same f_{vis}^0 . Since we are assuming an MSSM visible sector below the unification scale, one has to subject N_{vis}^i to the constraint that $f_{vis}^0(M_{\text{unif}}) \equiv \alpha_{unif}^{-1}(M_{\text{unif}}) \sim \mathcal{O}(25)$. The Yukawa couplings in these vacua arise from membrane instantons which connect singularities where chiral superfields are supported (if some singularities coincide, there could also be $\mathcal{O}(1)$ contributions). They are given by:

$$Y'_{\alpha\beta\gamma} = C_{\alpha\beta\gamma} e^{i2\pi \sum_i l_i^{\alpha\beta\gamma} z^i} \quad (10)$$

where $C_{\alpha\beta\gamma}$ is an $\mathcal{O}(1)$ constant and $l_i^{\alpha\beta\gamma}$ are integers. Factoring out the phases, the magnitude of the Yukawas can be schematically written as:

$$|Y| \sim |C| e^{-2\pi \vec{l} \cdot \vec{s}} \quad (11)$$

The normalized Yukawas differ from the above by factors corresponding to field redefinitions. Because of the exponential dependence on the moduli, it is natural to obtain a hierarchical structure of Yukawa couplings as is observed in nature. However, in general it is very difficult technically to compute the Yukawa couplings quantitatively. Therefore, for our phenomenological analysis, we will assume that the (normalized) Yukawa couplings are the same as those of the Standard Model. This is reasonable as in this work we are primarily interested in studying the effects of supersymmetry breaking and electroweak symmetry breaking.

Since the moduli have been stabilized, the F -terms of the moduli (F_i) and the meson fields (F_ϕ), which are the source of supersymmetry breaking, can be computed explicitly in terms of the microscopic constants. The expressions for F_i and F_ϕ in terms of these microscopic constants have been given explicitly in [1]. Since these F -terms and the quantities in (9) together determine the soft supersymmetry breaking parameters, it becomes possible to express all the soft parameters - gaugino masses, scalar masses, trilinears, μ and $B\mu$, in terms of the microscopic constants. Thus, given a particular G_2 -manifold one obtains a particular set of microscopic constants and thus a particular point in the MSSM parameter space. The set of microscopic constants consistent with the framework of G_2 compactifications and our assumptions thus defines a subset of the MSSM, which we call the G_2 -MSSM. How this works in practice should become clear in the following sections. Formulae (14), (20), (22), (32), (36) give the soft-breaking parameters at the unification scale, in terms of the microscopic constants.

Before moving on to discussing the phenomenology of the G_2 -MSSM vacua in detail, it is worth noting that realistic M theory vacua with a visible sector larger than the MSSM, will give rise to additional, different predictions for low energy phenomenology in general, and LHC signatures in particular. Therefore, the pattern of LHC signatures may help in distinguishing them. We hope to study this issue in the future.

II. G_2 -MSSM SOFT SUPERSYMMETRY BREAKING PARAMETERS AT M_{unif}

Phenomenologically relevant physics quantities are not sensitive to all of the microscopic constants. Instead, only certain parameters such as Q , P , C_2 and δ ⁴ as well as certain combinations of them, such as V_7 , $V_{\hat{Q}_{vis}}$ and P_{eff} are responsible for relevant physics quantities. These combinations have to satisfy various constraints which make the procedure of moduli stabilization in [1] valid and consistent. These conditions give phenomenologically interesting consequences. Four of the most important constraints are:

- The validity of the supergravity approximation requires $V_7 > 1$. We call this the “weak supergravity constraint”. In its “strong” form, one could require all geometric moduli s_i to be greater than unity. The supergravity constraint is required because our solutions can be trusted only in this regime. There could be different solutions in other regimes beyond the supergravity approximation, but little is known about them.
- Since current observations indicate a strong evidence for a dS vacuum with a tiny cosmological constant, the vacua are required to have positive cosmological constant. In addition, the microscopic constants are required to be such that the cosmological constant can be tuned to be very small. We call this the “dS vacuum constraint”.
- Since it is known that, for an MSSM visible sector $\alpha_{\text{unif}}^{-1} \sim \mathcal{O}(25)$, this implies $V_{\hat{Q}_{vis}} \sim \alpha_{\text{unif}}^{-1} \sim \mathcal{O}(25)$. We call this the “unified coupling constraint”.
- $M_{11} > M_{\text{unif}} \sim \mathcal{O}(10^{16} \text{ GeV})$, which is required to make intrinsic M theory corrections to gauge couplings and other parameters negligible at and below M_{unif} . We call this the “unification scale constraint”.

A discussion of parameters compatible with the “supergravity constraint”, “dS vacuum constraint” and the “unified coupling constraint” is given in Appendix B. In section IV, it will be shown that the “unification scale” constraint can be naturally satisfied within this framework. We will now proceed to discussing the gravitino mass and the soft supersymmetry breaking parameters assuming that sets of parameters can be found such that all the above constraints are satisfied.

A. Gravitino Mass

The bare gravitino mass in $\mathcal{N} = 1$ supergravity can be computed as follows:

$$m_{3/2} \equiv m_p^{-2} e^{\frac{K}{2m_p^2}} |W| \quad (12)$$

This quantity plays an important role in gravity mediated models of supersymmetry breaking and sets the typical mass scale for couplings in the supergravity Lagrangian. It is therefore useful to compute this quantity in detail in the G_2 -MSSM.

⁴ This is defined in section II C 1.

As explained earlier, $|W|$ is generated by strong gauge dynamics in the two hidden sectors, $W_{1,2} \sim (\Lambda_{1,2})^3$. This implies that the gravitino mass can be schematically expressed as:

$$m_{3/2} \sim \frac{\Lambda^3}{m_p^2} \quad (13)$$

up to some factors of volume coming from the Kähler potential in (12). More precisely, the gravitino mass is:

$$\begin{aligned} m_{3/2} &= m_p \frac{e^{\phi_0^2/2}}{8\sqrt{\pi}V_7^{3/2}} C_2 \left| P\phi_0^{-\frac{2}{P}} - Q \right| e^{-\frac{P_{\text{eff}}}{Q-P}} \\ &\approx m_p \frac{e^{\phi_0^2/2}}{8\sqrt{\pi}V_7^{3/2}} C_2 |Q - P| e^{-\frac{P_{\text{eff}}}{Q-P}} \end{aligned} \quad (14)$$

where in the last line, $\phi_0 \approx 1$ is used. The exponential part in the equation is roughly Λ_{cond}^3 in units of m_p . As seen from above, the gravitino mass is effectively determined by four parameters: $\{P_{\text{eff}}, Q - P, V_7 \text{ and } C_2\}$. For $Q - P = 3$, the gravitino mass can be well approximated by

$$m_{3/2} = (708 \text{ TeV}) \times \left(C_2 V_7^{-3/2} e^{-\frac{P_{\text{eff}} - 83}{3}} \right) \quad (15)$$

For the case of zero cosmological constant $P_{\text{eff}} = 83$ ⁵, the exponential is unity and the gravitino mass is bounded from above by 708 TeV. As will be seen later, V_7 turns out to be typically in the range 10-100, implying that $m_{3/2}$ naturally lies between 10 and 100 TeV. If one allows a dS minimum with a large cosmological constant, P_{eff} can be smaller than 84 and the gravitino mass can become larger. For larger values of $Q - P$, the P_{eff} required to tune the cosmological constant (see eqn. (8)) is smaller. For example, for $Q - P = 4$, $P_{\text{eff}} = 28$ is required. Since the gravitino mass is exponentially sensitive to P_{eff} (as seen from (15)), the gravitino mass for $Q - P > 3$ turns out to be much larger than the TeV scale. This is the reason for mainly considering the case $Q - P = 3$.

B. Scalars and Trilinears at M_{unif}

The general expressions for the *un-normalized* scalar masses and trilinear parameters are given by [14]:

$$\begin{aligned} m_{\tilde{\alpha}\beta}^2 &= (m_{3/2}^2 + V_0) \tilde{K}_{\tilde{\alpha}\beta} - e^{\hat{K}} F^{\tilde{m}} (\partial_{\tilde{m}} \partial_n \tilde{K}_{\tilde{\alpha}\beta} - \partial_{\tilde{m}} \tilde{K}_{\tilde{\alpha}\gamma} \tilde{K}^{\gamma\tilde{\delta}} \partial_n \tilde{K}_{\tilde{\delta}\beta}) F^n \\ A'_{\alpha\beta\gamma} &= \frac{\hat{W}^*}{|\hat{W}|} e^{\hat{K}} F^m [\hat{K}_m Y'_{\alpha\beta\gamma} + \partial_m Y'_{\alpha\beta\gamma} - (\tilde{K}^{\delta\bar{\rho}} \partial_n \tilde{K}_{\bar{\rho}\alpha} Y'_{\alpha\beta\gamma} + \alpha \leftrightarrow \gamma + \alpha \leftrightarrow \beta)] \end{aligned} \quad (16)$$

In order to determine physical implications, however, one has to canonically normalize the visible matter Kähler potential $K_{\text{visible}} = \tilde{K}_{\tilde{\alpha}\beta} \Phi^{\tilde{\alpha}} \Phi^{\beta} + \dots$, which is achieved by introducing a normalization matrix \mathcal{U} :

$$\Phi \rightarrow \mathcal{U} \cdot \Phi, \quad \text{s.t.} \quad \mathcal{U}^\dagger \tilde{K} \mathcal{U} = 1. \quad (17)$$

⁵ The upper limit $P_{\text{eff}} = 84$ obtained in the zeroth order of the $1/V_{\tilde{Q}_{\text{vis}}}$ approximation is modified to $P_{\text{eff}} = 83$ after taking higher order corrections into account.

The \mathcal{U} s are themselves only defined up to a unitary transformation, i.e. $\mathcal{U}' = \mathcal{U} \cdot \mathcal{N}$ is also an allowed normalization matrix if \mathcal{N} is unitary. The *normalized* scalar masses and trilinears can then be written formally as:

$$\begin{aligned} m_{\bar{\alpha}\beta}^2 &= (\mathcal{U}^\dagger \cdot m'^2 \cdot \mathcal{U})_{\bar{\alpha}\beta} \\ \tilde{A}_{\alpha\beta\gamma} &= \mathcal{U}_{\alpha\alpha'} \mathcal{U}_{\beta\beta'} \mathcal{U}_{\gamma\gamma'} A'_{\alpha'\beta'\gamma'} \end{aligned} \quad (18)$$

More precisely, the scalar masses can be written as:

$$\begin{aligned} m_{\bar{\alpha}\beta}^2 &= (m_{3/2}^2 + V_0) \delta_{\bar{\alpha}\beta} - \mathcal{U}^\dagger \Gamma_{\bar{\alpha}\beta} \mathcal{U} \\ \Gamma_{\bar{\alpha}\beta} &\equiv e^{\hat{K}} F^{\bar{m}} (\partial_{\bar{m}} \partial_n \tilde{K}_{\bar{\alpha}\beta} - \partial_{\bar{m}} \tilde{K}_{\bar{\alpha}\gamma} \tilde{K}^{\gamma\bar{\delta}} \partial_n \tilde{K}_{\bar{\delta}\beta}) F^n \end{aligned} \quad (19)$$

Thus, when the cosmological constant has been tuned to be small, the scalar masses generically have a flavor universal and flavor diagonal contribution equal to $m_{3/2}^2$ from the first term in (19) and a flavor non-universal and flavor non-diagonal contribution from the second term in (19). In order to estimate the size of the non-universal and non-diagonal contributions, one has to know about the moduli and meson dependence of the visible sector Kähler metric. This dependence of the matter Kähler metric is notoriously difficult to compute in generic string and M theory vacua, and the vacua under study here are no exception. Therefore, it is only possible to proceed by making reasonable assumptions. Under our assumptions about the meson field kinetic term, the only contribution to the non-universal and non-diagonal terms in (19) comes from the F terms of the moduli F_i . Since $F_i \ll F_\phi$, the non-universal contributions are negligible compared to the universal and diagonal contribution. Thus,

$$m_{\bar{\alpha}\beta}^2 \approx m_{3/2}^2 \delta_{\bar{\alpha}\beta} \quad (20)$$

This implies that flavor changing neutral currents (FCNCs) are adequately suppressed. The fact that the scalar masses are roughly equal to the gravitino mass can be traced to the non-sequestered nature of the Kähler potential in (9). In the absence of fluxes, the G_2 compactifications considered here do not have any warping, which implies that one generically does not have sequestering in these compactifications [15]. Since the scalars are heavy and also flavor universal at leading order, we expect that no significant signals should occur for observables from loops with sleptons or squarks, in particular for rare flavor violating decays such as $\mu \rightarrow e\gamma$, $K \rightarrow \pi\nu\bar{\nu}$, $b \rightarrow s\gamma$, etc, and also no significant signal for $g_\mu - 2$.

The computation of the trilinears also simplifies under the above assumptions. Again, since the un-normalized Yukawa couplings and the visible sector Kähler metric are not expected to depend on the meson field, the dominant contribution to the trilinears comes from the first term in the expression for trilinears in (IIB). Thus, one has:

$$A'_{\alpha\beta\gamma} \approx \frac{\hat{W}^\star}{|\hat{W}|} e^{\hat{K}} F^\phi \hat{K}_\phi Y'_{\alpha\beta\gamma} \quad (21)$$

This implies that the normalized trilinear parameters are:

$$\begin{aligned} \tilde{A}_{\alpha\beta\gamma} &\approx (\mathcal{U}_{\alpha\alpha'} \mathcal{U}_{\beta\beta'} \mathcal{U}_{\gamma\gamma'}) \frac{\hat{W}^\star}{|\hat{W}|} e^{\hat{K}} F^\phi \hat{K}_\phi Y'_{\alpha'\beta'\gamma'} \\ &\approx e^{\hat{K}/2} F^\phi \hat{K}_\phi Y_{\alpha\beta\gamma} \\ &\approx e^{-i\gamma_W} 1.48 m_{3/2} Y_{\alpha\beta\gamma} \end{aligned} \quad (22)$$

Here we have used the fact that the normalized Yukawa couplings are given by $Y_{\alpha\beta\gamma} = \frac{\hat{W}^*}{|W|} e^{\hat{K}/2} (\mathcal{U}_{\alpha\alpha'} \mathcal{U}_{\beta\beta'} \mathcal{U}_{\gamma\gamma'}) Y'_{\alpha'\beta'\gamma'}$ and that $e^{\hat{K}/2} F^\phi \hat{K}_\phi \approx e^{-i\gamma_W} \sqrt{3} \phi_0 m_{3/2} \approx e^{-i\gamma_W} 1.48 m_{3/2}$ [1]. The reduced normalized trilinear parameters have a particularly simple form:

$$A_{\alpha\beta\gamma} \equiv \frac{\tilde{A}_{\alpha\beta\gamma}}{Y_{\alpha\beta\gamma}} \approx e^{-i\gamma_W} 1.48 m_{3/2} \quad (23)$$

Thus, we see that in G_2 -MSSM vacua, the scalar masses and trilinears are generically of $\mathcal{O}(m_{3/2})$.

C. Gaugino Masses at M_{unif}

We now turn to gaugino masses. The computation of gaugino masses depends less on our knowledge of the matter Kähler potential, therefore it is possible to obtain quite detailed formulae. In G_2 vacua the tree-level gaugino masses are suppressed relative to the gravitino mass unlike the scalars and trilinears. Therefore, other contributions such as those from anomaly mediation and those from threshold effects arising from integrating out heavy fields can be important. Schematically, one can write

$$M_a(\mu) = M_a^{\text{tree}}(\mu) + M_a^{\text{AMSB}}(\mu) + M_a^{\text{thres}}(\mu) \quad (24)$$

In the following we wish to compute each contribution at the unification scale M_{unif} . We study the case when the low energy spectrum is that of the MSSM. As mentioned in section IB, for concreteness we will assume an $SU(5)$ GUT group broken to the MSSM by a discrete choice of Wilson lines for concreteness. This gives rise to a pair of Higgs triplets whose effects should be properly taken into account. For the case of a different GUT group breaking to the MSSM by Wilson lines, there would be similar heavy particles whose effects should be taken into account. As we will see, the results obtained will be the same for all GUT groups as long as the low energy spectrum is that of the MSSM.

1. Tree-level suppression of Gaugino masses

The tree-level gaugino masses at the scale μ are given by [14]:

$$M_a^{\text{tree}}(\mu) = \frac{g_a^2(\mu)}{8\pi} \left(\sum_{m,n} e^{\hat{K}/2} K^{m\bar{n}} F_{\bar{n}} \partial_m \text{Im} f_a^0 \right) \quad (25)$$

$$= \frac{g_a^2(\mu)}{8\pi} \sum_{i=1}^N e^{\hat{K}/2} K^{i\bar{i}} F_{\bar{i}} N_i^{\text{vis}}. \quad (26)$$

where f_a^0 is the tree-level gauge kinetic function of the a^{th} gauge group. As explained earlier, the tree-level gauge kinetic function f_a^0 of the three gauge groups in the MSSM are the same ($= f_{\text{vis}}^0$) because of the underlying GUT structure. The tree-level gaugino mass at the unification scale can then be computed in terms of microscopic parameters. The details are provided in section VIIIA of [1]. Here, we write down the result:

$$M_a^{\text{tree}}(M_{\text{unif}}) \approx -\frac{e^{-i\gamma_W} \eta}{P_{\text{eff}}} \left(1 + \frac{2}{\phi_0^2 (Q - P)} + \mathcal{O}(P_{\text{eff}}^{-1}) \right) m_{3/2}$$

where $\alpha_{\text{unif}}^{-1} = \text{Im}(f_{\text{vis}}^0) + \delta$; $\eta = 1 - \frac{\delta}{\alpha_{\text{unif}}^{-1}}$ (27)

where γ_W arises from the phase of the F -term. The phases of the soft parameters will be discussed in detail in section IID. δ corresponds to threshold corrections to the (unified) gauge coupling and will be discussed more in section IV. As seen from above, gaugino masses are suppressed by P_{eff} relative to gravitino mass. This property is independent of the details of the Kähler potential for ϕ and the form of V_7 .

Because the MSSM is obtained by the breaking of a GUT group by Wilson lines, the gauge couplings of the MSSM gauge groups are unified at the unification scale giving rise to a common $\text{Im}(f_{vis}^0)$. This implies that the tree level gaugino masses at M_{unif} are also unified. In particular, for $Q - P = 3$ with a vanishing cosmological constant ($P_{\text{eff}} = 83$) and the Kähler potential given by (3), one has an explicit expression for the gaugino mass:

$$\begin{aligned} M_a^{\text{tree}}(M_{\text{unif}}) &\approx -\frac{e^{-i\gamma_W}}{83}\eta\left(1 + \frac{2}{3\phi_0^2} + \frac{7}{83\phi_0^2}\right) \times m_{3/2} \\ &\approx e^{-i\gamma_W} \eta 0.024 m_{3/2} \end{aligned} \quad (28)$$

Here we have used the fact that for $Q - P = 3$, $\phi_0^2 \approx 0.73$. As explained in section IA, a large P_{eff} is required for the validity of our solutions. Therefore, the parametric dependence on P_{eff}^{-1} in Eq.(27) indicates a large suppression in gaugino masses. The precise numerical value of the suppression may change if one considers a more general form of the matter Kähler potential since then the numerical factor multiplying P_{eff}^{-1} in (27) may change in general. However as long as the couplings for higher order terms in the matter Kähler potential such as $(\bar{\phi}\phi)^2$ are sufficiently small, a large numerical suppression is generic. In our analysis henceforth, we will assume that to be the case.

From a physical point of view, the suppression of gaugino masses is directly related to the fact that the F -terms of moduli F_i (in Planck units) are suppressed compared to $m_{3/2}$ and that the gauge kinetic function f_a^0 in (25) only depends on the moduli s_i . This implies that the F -term of the meson field does not contribute in (27). It is also useful to compare the above result for the tree-level suppression of gaugino masses in G_2 dS vacua with that of Type IIB dS vacua. In KKLT and large volume type IIB compactifications, the moduli F terms also vanish in the leading order leading to a suppression of tree-level gaugino masses, although for a different reason - the flux contribution to the moduli F terms cancels the contribution coming from the non-perturbative superpotential [16]. Another difference is that the subleading contributions in those Type IIB vacua are suppressed by the inverse power of the volume of the compactification manifold. Note that a large associative three-cycle on a G_2 manifold ($V_{\hat{Q}_{vis}}$) does *not* translate into a large volume compactification manifold. So, it is possible for G_2 vacua to have a large $V_{\hat{Q}_{vis}}$ while still having a high compactification scale.

2. Anomaly contributions

Since the tree-level gaugino mass is suppressed, the anomaly mediated contributions become important and should be included. In our framework they are not suppressed. The general expression for the

anomaly contribution to gaugino masses at scale μ is written as [17]:

$$M_a^{anom}(\mu) = -\frac{g_a^2(\mu)}{16\pi^2} \left(-(3C_a - \sum_i C_a^i) e^{\hat{K}/2} W^* + (C_a - \sum_i C_a^i) e^{\hat{K}/2} F^m \hat{K}_m \right. \\ \left. + 2 \sum_i (C_a^i e^{\hat{K}/2} F^m \partial_m \ln(\tilde{K}_i)) \right), \quad (29)$$

where i runs over all the MSSM chiral fields. Again, since the Kähler metric for visible sector matter fields is expected to be independent of the meson field, the third term in (29) is much smaller than the first two. Thus, the anomaly mediated contribution can be simplified:

$$M_a^{anom}(M_{\text{unif}}) \approx -\frac{e^{-i\gamma_W} g_a^2}{16\pi^2} \left[b_a + \left(1 + \frac{2}{(Q-P)\phi_0^2} + \frac{7}{\phi_0^2 P_{\text{eff}}} \right) \left(-b'_a (\phi_0^2 + \frac{7}{P_{\text{eff}}}) \right) \right] m_{3/2} \quad (30)$$

$$b_a \equiv -(3C_a - \sum_i C_a^i); \quad b'_a \equiv -(C_a - \sum_i C_a^i)$$

As seen from (30), the anomaly mediated contributions are not universal. Since the anomaly mediated contributions are numerically comparable to the tree-level contributions, the gaugino masses will be non-universal at the unification scale. That the tree-level and anomaly mediated contributions are similar in size seems to be accidental – one is suppressed by $1/P_{\text{eff}} = 1/83$, the other by the loop factor $1/16\pi^2$, and these factors are within a factor of two of each other.

3. The complete Gaugino masses

In principle, there can also be threshold corrections to gaugino masses from high scale physics and it is important to take them into account. In general, a threshold correction to the gaugino masses at a scale M_{th} is induced by a threshold correction to gauge couplings by the following expression [18]:

$$\Delta M_a = g_a^2(M_{th}) F^I \partial_I (\Delta f_a^{thresh}) \quad (31)$$

In these M theory compactifications, possible threshold corrections at scales $\leq M_{\text{unif}}$ can arise from the following:

- Generic heavy M theory excitations Ψ of $\mathcal{O}(M_{11})$ - $\Delta f_a^{M \text{ theory}}$.
- 4D particles in the GUT multiplet with mass $\approx M_{\text{unif}}$ - $\Delta f_a^{T, \tilde{T}}$.
- Kaluza-Klein (KK) excitations of $\mathcal{O}(M_{\text{unif}})$ - Δf_a^{KK} .

It turns out that threshold corrections to the gauge couplings from KK modes are constants [7]. Therefore, from (31), they will *not* give rise to any threshold correction to the gaugino masses. As explained in appendix C, corrections from generic heavy M theory states T with mass $\sim M_{11}$ as well as from 4D heavy GUT particles with mass ($\sim M_{\text{unif}}$) (such as Higgs triplets in $SU(5)$), to the gaugino masses are also negligible. So, the complete gaugino mass can be approximately written as:

$$M_a(M_{\text{unif}}) \approx -\frac{e^{-i\gamma_W}}{4\pi(\text{Im}(f^0) + \delta)} \left\{ b_a + \left(\frac{4\pi \text{Im}(f^0)}{P_{\text{eff}}} - b'_a \phi_0^2 \right) \left(1 + \frac{2}{\phi_0^2(Q-P)} + \frac{7}{\phi_0^2 P_{\text{eff}}} \right) - \frac{7b'_a}{P_{\text{eff}}} \right\} m_{3/2}$$

where $b_1 = 33/5$, $b_2 = 1.0$, $b_3 = -3.0$, $b'_1 = -\frac{33}{5}$, $b'_2 = -5.0$, $b'_3 = -3$. (32)

The above analytical expression for gaugino masses is true up to the first subleading order in the $1/V_{\hat{Q}}$ expansion. In general, the full gaugino mass at the unification scale (32) depends on the parameters $\{\text{Im}(f^0), Q - P, \delta, V_7, P_{\text{eff}} \text{ and } C_2\}$. For the phenomenologically interesting case with $Q - P = 3$ and $P_{\text{eff}} = 83$, there are effectively only four parameters: $\{\text{Im}(f^0), \delta, V_7, C_2\}$. As seen from (15), $m_{3/2}$ is determined from the last two parameters - V_7 and C_2 . Therefore, the ratio of the gaugino masses to the gravitino mass for the phenomenologically interesting case of $Q - P = 3, P_{\text{eff}} = 83$ just depends on the parameters $\text{Im}(f^0)$ and δ which are subject to the constraint $\alpha_{\text{unif}}^{-1} = \text{Im}(f^0) + \delta \approx \mathcal{O}(25)$ (see section IB). The gaugino masses are plotted as a function of δ in Figure 1 for $m_{3/2} = 20$ TeV. Notice that M_2 and

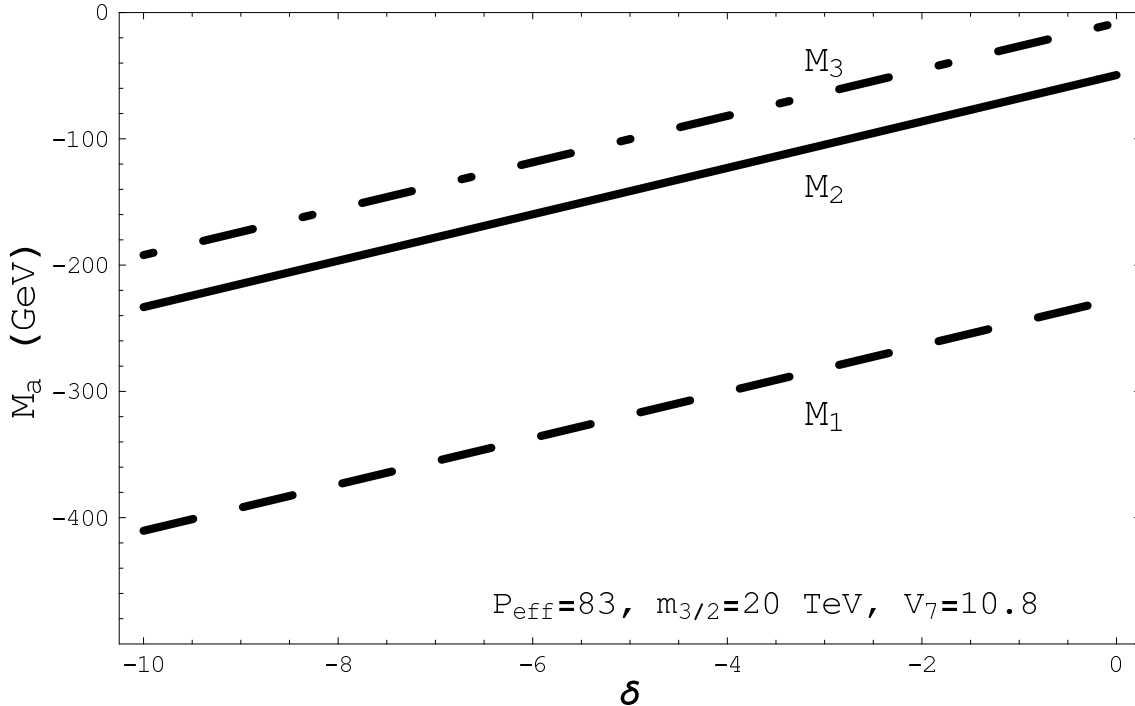


FIG. 1: The absolute value of the gaugino masses at the unification scale are plotted as functions of threshold correction to the gauge kinetic function δ for $Q - P = 3, P_{\text{eff}} = 83, m_{3/2}=20$ TeV and $\alpha_{\text{unif}}^{-1} = 26.5$.

M_3 are smaller than M_1 by a factor of few at the unification scale. However, as will be seen promptly, the gluino will still turn out to be the heaviest gaugino because of RG effects.

D. Phases, μ and $B\mu$.

Until now, we have not looked at the phases in detail. In general, phases may have non-trivial phenomenological consequences. To count the number of phases, it is helpful to go back to the expression for the superpotential:

$$W = A_1 \phi^\alpha e^{ib_1 f} + A_2 e^{ib_2 f}; \quad \alpha \equiv -\frac{2}{P} \quad (33)$$

where A_1 and A_2 and ϕ are complex variables with phases say $(\varphi_1, \varphi_2, \theta)$. However the relative phase between the first and second terms is fixed by the minimization of axions,

$$\begin{aligned}\sin\left((b_1 - b_2)\vec{N} \cdot \vec{t} + \alpha\theta + \varphi_1 - \varphi_2\right) &= 0 \\ \cos\left((b_1 - b_2)\vec{N} \cdot \vec{t} + \alpha\theta + \varphi_1 - \varphi_2\right) &= -1\end{aligned}\quad (34)$$

This can be seen in the Eq. (102) and (103) in [1]. The only phase left is $\gamma_W \equiv b_2 \vec{N} \cdot \vec{t} + \varphi_2$.

Let's start with the calculation of phases for the gauginos. From the definition of the tree-level gaugino masses in (25), $M_a \propto F_{\bar{m}} \equiv \partial_{\bar{m}} \bar{W} + (\partial_{\bar{m}} K) \bar{W}$. Thus, the phase of the tree-level gaugino mass is $-\gamma_W$ (even though the Kähler potential appears in the expression for gaugino masses, it is real and hence does not contribute to the phase). There is an additional contribution from anomaly-mediation which includes terms proportional to either \bar{W} or $\bar{F}_{\bar{m}}$, leading to the same phase $-\gamma_W$ as seen in (30). Thus, the phase of the gauginos is $\phi_{M_a} = -\gamma_W$.

The Yukawa couplings also have phases in general. As mentioned in section IB, in our analysis we have not attempted to explain the origin of Yukawa couplings and have assumed that the Yukawa couplings are the same as that in the Standard-Model. This effectively means that the Yukawas just contain one non-trivial phase, the one which is present in the CKM matrix.

The phases of supersymmetry breaking trilinear couplings can also be computed in terms of γ_W . From (22), $A_{\alpha\beta\gamma} \propto F^\phi \hat{K}_\phi = K^{\phi\bar{\phi}} F_{\bar{\phi}} \hat{K}_\phi$. Then one can show that $\hat{K}_\phi = \phi_0 e^{-i\theta}$ and $F_{\bar{\phi}} \sim e^{i\theta - i\gamma_W}$. Therefore, the overall phase for the trilinear coupling is simply $\phi_A = -\gamma_W$, the same as that for gaugino masses.

The μ and $B\mu$ parameters (with $\mu' = 0$ ⁶) can be written as [14]:

$$\begin{aligned}\mu &= \left(e^{i\gamma_W} m_{3/2} Z - e^{\hat{K}/2} F^{\bar{m}} \partial_{\bar{m}} Z \right) (\tilde{K}_{H_u} \tilde{K}_{H_d})^{-1/2} \\ B\mu &= \left\{ (2m_{3/2}^2 + V_0) Z - e^{i\gamma_W} m_{3/2} e^{\hat{K}/2} F^{\bar{m}} \partial_{\bar{m}} Z + e^{i\gamma_W} m_{3/2} e^{\hat{K}/2} F^m [\partial_m Z - Z \partial_m \ln(\tilde{K}_{H_u} \tilde{K}_{H_d})] \right. \\ &\quad \left. - e^{\hat{K}} F^{\bar{m}} F^n [\partial_{\bar{m}} \partial_n Z - \partial_{\bar{m}} Z \partial_n \ln(\tilde{K}_{H_u} \tilde{K}_{H_d})] \right\} (\tilde{K}_{H_u} \tilde{K}_{H_d})^{-1/2}\end{aligned}\quad (35)$$

where Z is the Higgs bilinear coefficient and \tilde{K} is the Kähler metric for the Higgs fields. Z is a complex-valued function of all hidden sector chiral fields in general. Therefore, in general both μ and $B\mu$ have independent phases. However, with the reasonable assumption that the visible sector Kähler metric and the Higgs bilinear coefficient Z are independent of the meson field ϕ , one can make simplifications. Combining the above with the fact that $F^i \ll F^\phi$, one has the following approximation:

$$\begin{aligned}\mu &\approx e^{i\gamma_W} m_{3/2} Z (\tilde{K}_{H_u} \tilde{K}_{H_d})^{-1/2} \equiv e^{i\gamma_W} m_{3/2} Z_{\text{eff}} \\ B\mu &\approx (2m_{3/2}^2 + V_0) Z (\tilde{K}_{H_u} \tilde{K}_{H_d})^{-1/2} \equiv (2m_{3/2}^2 + V_0) Z_{\text{eff}}\end{aligned}\quad (36)$$

Therefore, the phases of μ and $B\mu$ are:

$$\begin{aligned}\phi_\mu &\approx \gamma_W + \gamma_Z \\ \phi_{B\mu} &\approx \gamma_Z\end{aligned}\quad (37)$$

⁶ which is favored by the motivation of the solution to the doublet-triplet splitting problem[13].

To summarize, with some reasonable assumptions, at leading order in our analysis, all soft terms depend on only two phases - γ_W and γ_Z . This means that the reparameterization invariant phases given by $\phi_\mu + \phi_A - \phi_{B\mu}$ and $\phi_\mu + \phi_{M_a} - \phi_{B\mu}$ [19] vanish, implying that within the above assumptions there are no non-trivial CP-violating phases beyond that in the Standard-Model at the unification scale. It is possible to do a global phase transformation of the superpotential without affecting physical observables. Thus, one can choose a basis in which γ_W vanishes. The reparameterization invariant combinations above still give the same result.

RG evolution to low scales can lead to additional effects. There is a finite threshold correction to the bino and wino mass parameters from the Higgs-Higgsino loop at low-scales which is proportional to μ (more about this in the next section). The correction thus depends on the phase ϕ_μ making the phase of the low scale bino and wino mass parameters different from $-\gamma_W$ in general. Therefore, the reparameterization invariant combinations $\phi_\mu + \phi_{M_a} - \phi_\mu$ for $a = 1, 2$ will not vanish in general at low scales giving rise to non-trivial phases. In the basis in which γ_W vanishes, these non-trivial phases will depend on γ_Z . At present, it is not possible to reliably compute γ_Z from first principles. Therefore, in our analysis below, we will only study situations with $\gamma_Z = 0$ or π ⁷. This corresponds to μ being positive or negative respectively. We hope to study non-trivial phases in more detail in the future.

E. Summary of the G_2 -MSSM at the Unification scale.

In summary, the soft-susy breaking parameters at the unification scale in the G_2 -MSSM are given by equations (20), (23), (32) and (36) (with z_{eff} order one). The microscopic constants which are determined by the G_2 -manifold are subject to the constraints discussed at the beginning of section II. We now turn to the detailed discussion of renormalising the masses and couplings down to the Electroweak scale.

III. SUPERPARTNER SPECTRUM AT M_{EW} AND ELECTROWEAK SYMMETRY BREAKING

As seen in previous section, the scalar and Higgsino masses at M_{unif} are close to that of the gravitino. This has to be larger than $\gtrsim 10$ TeV in order to evade the LEP II chargino bound because of the large suppression of the gaugino masses relative to the gravitino. In addition, a gravitino mass of $\gtrsim 10$ TeV is also required to mitigate the moduli and gravitino problems.

In order to connect to low-energy physics, one has to RG evolve the soft supersymmetry breaking parameters from M_{unif} to the electroweak scale. It turns out that RG evolution increases the masses of the first and second generation squarks and sleptons. However, since the increase is mostly proportional to the gaugino masses [20] which are much smaller than the high-scale sfermion mass, the masses of the first and second generations squarks and sleptons are still of $\mathcal{O}(m_{3/2})$. The masses of the third generation squarks and sleptons - stops, sbottoms and staus are also affected non-trivially by the trilinear parameters (again of $\mathcal{O}(m_{3/2})$) because of their larger Yukawa couplings [20]. In particular, the lightest stop (\tilde{t}_1) becomes much lighter than the other sfermions (even though still considerably heavier than the gauginos).

⁷ This is in the basis in which $\gamma_W = 0$.

Finally, the μ parameter, which determines the masses of the Higgsinos, does not change much during RG evolution because of the non-renormalization theorem⁸. So, the μ parameter at the electroweak scale is also of $\mathcal{O}(m_{3/2})$.

Because of the large hierarchy in the spectrum, it is convenient to work in an effective theory with the heavy fields (scalars and Higgsinos) integrated out at their characteristic scale ($M_s \sim 10 - 100$ TeV). The low energy effective theory below M_s only contains the light gauginos and the SM particles. Therefore it is very important to compute the masses of gauginos as these are the only light new states predicted by the theory⁹. To take into account the threshold effects of these heavy states on the gaugino masses, we follow the ‘match and run’ procedure which is a good approximation when $M_a \ll M_s$. In this paper, we use a one-loop two-stage RGE running with a tree-level matching at the scale M_s . All other thresholds are calculated using the exact one-loop results.

A. Gaugino Masses at M_{EW}

The weak scale gaugino mass parameters at one-loop can be related to those at unification scale by a RG evolution factor K_a as follows:

$$M_a(M_{\text{weak}}) = K_a M_a(M_{\text{unif}}) \quad (38)$$

The RG evolution factors K_a are given by:

$$K_a = \left(\frac{\alpha_a^s}{\alpha_{unif}^s} \right) \left(\frac{\alpha_a^{EW}}{\alpha_a^s} \right)^{\tilde{b}_a^{SM}/b_a^{SM}} \quad (39)$$

where \tilde{b}_a ’s and b_a ’s are the β function coefficients of the gaugino masses and gauge couplings respectively:

$$\begin{aligned} \tilde{b}_1^{SM} &= 0, & \tilde{b}_2^{SM} &= -6, & \tilde{b}_3^{SM} &= -9 \\ b_1^{SM} &= \frac{41}{10}, & b_2^{SM} &= -\frac{11}{6}, & b_3^{SM} &= -5. \end{aligned} \quad (40)$$

α_a^s and α_a^{EW} are the gauge couplings at the decoupling scale M_s and the electro-weak scale M_{EW} respectively, which can be expressed as

$$\begin{aligned} (\alpha_a^s)^{-1} &= \alpha_{unif}^{-1} + \frac{b_a}{2\pi} \ln \left(\frac{M_{unif}}{M_s} \right) \\ (\alpha_a^{EW})^{-1} &= (\alpha_a^s)^{-1} + \frac{b_a^{SM}}{2\pi} \ln \left(\frac{M_s}{M_{EW}} \right) \end{aligned} \quad (41)$$

As an example, for $\alpha_{unif} = 1/26.5$, for M_s varying from 10 TeV to 100 TeV, the corresponding RG factors are

$$K_1 \approx 0.47 - 0.49, \quad K_2 \approx 0.99 - 1.08, \quad K_3 \approx 3.7 - 4.6. \quad (42)$$

⁸ it only suffers from wave-function renormalization.

⁹ except possibly the lightest stop.

Notice that the RG evolution factor K_3 is quite large for a large M_s . This prevents the gluino becoming the LSP even though the gluino mass M_3 is typically small at the unification scale.

Once the running masses of the gauginos at the low scale are calculated, their pole mass can be obtained by adding weak scale threshold corrections. For general MSSM parameters, they are given in [21]. In our ‘match and run’ procedure, the threshold corrections of heavy scalars and Higgsinos are automatically included except some finite terms which are usually small and negligible. However, since in our case the Higgsino mass μ is of order $m_{3/2}$, the finite threshold correction cannot be neglected and is given by [21, 22, 23]:

$$\begin{aligned}\Delta M_{1,2}^{\text{finite}} &\approx -\frac{\alpha_{1,2}}{4\pi} \frac{\mu \sin(2\beta)}{(1 - \frac{\mu^2}{m_A^2})} \ln\left(\frac{\mu^2}{m_A^2}\right) \\ &\approx \frac{\alpha_{1,2}}{4\pi} \mu = \frac{\alpha_{1,2}}{4\pi} z_{\text{eff}} m_{3/2}\end{aligned}\tag{43}$$

In the second line, we have used the fact that $\frac{\mu^2}{m_A^2} \sim 1$ so that the logarithm can be expanded. In addition, since μ does not change much in the RG evolution, it is a good approximation to use its high scale value $\mu \equiv z_{\text{eff}} m_{3/2}$. This quantum correction proportional to μ will shift the bino (M_1) and wino (M_2) masses up or down depending on the sign of μ . This will most significantly affect M_2 as it is typically the lightest gaugino at M_{unif} . Therefore it could potentially affect the identity of the LSP. For the case with gravitino mass $m_{3/2} \sim 30$ TeV, this finite correction to M_2 is roughly 100 GeV, which may even dominate over the tree-level mass. This large correction to $M_{1,2}$ is not surprising since the low energy effective theory is non-supersymmetric and there is no symmetry to protect the gaugino masses from finite radiative corrections.

It is important to notice that since the above correction is linear in μ , it is *sensitive* to the sign of μ . More generally, as was explained in section IID, μ can have a different non-trivial phase from the gaugino masses which share a common phase. So the inclusion of the finite threshold correction can eventually lead to non-trivial phases for the reparameterization invariant combinations. This can change the spectrum eigenvalues, leading to observable effects for colliders, for Dark Matter relic density, Dark Matter detection and for Electric Dipole Moments (EDMs). As mentioned in section IID, it is unfortunately not possible at present to compute the effects of these phases reliably, so we will only consider the effects of positive and negative μ (corresponding to $\gamma_Z = 0$ and π in our analysis respectively¹⁰). This leads to two different plots as shown in Figure 2.

Of course, one also has to include weak scale threshold corrections from gaugino-gauge-boson loops. These are especially important for the gluino mass:

$$\Delta M_3^{\text{rad}} = \frac{3g_3^2}{16\pi^2} \left(3 \ln \left(\frac{M_{\text{EW}}^2}{M_3^2} \right) + 5 \right) M_3\tag{44}$$

For M_3 not much heavier than M_{EW} , there is a substantial correction of at least $3\alpha_3 M_3$.

We calculated the gaugino mass at the Weak scale, with all these corrections taken into account. As mentioned before, the hierarchy of gaugino masses is most sensitive to δ and P_{eff} . In order to obtain realistic phenomenology, we choose $Q - P = 3$ and tune the cosmological constant to obtain $P_{\text{eff}} = 83$.

¹⁰ This is in the basis in which $\gamma_W = 0$.

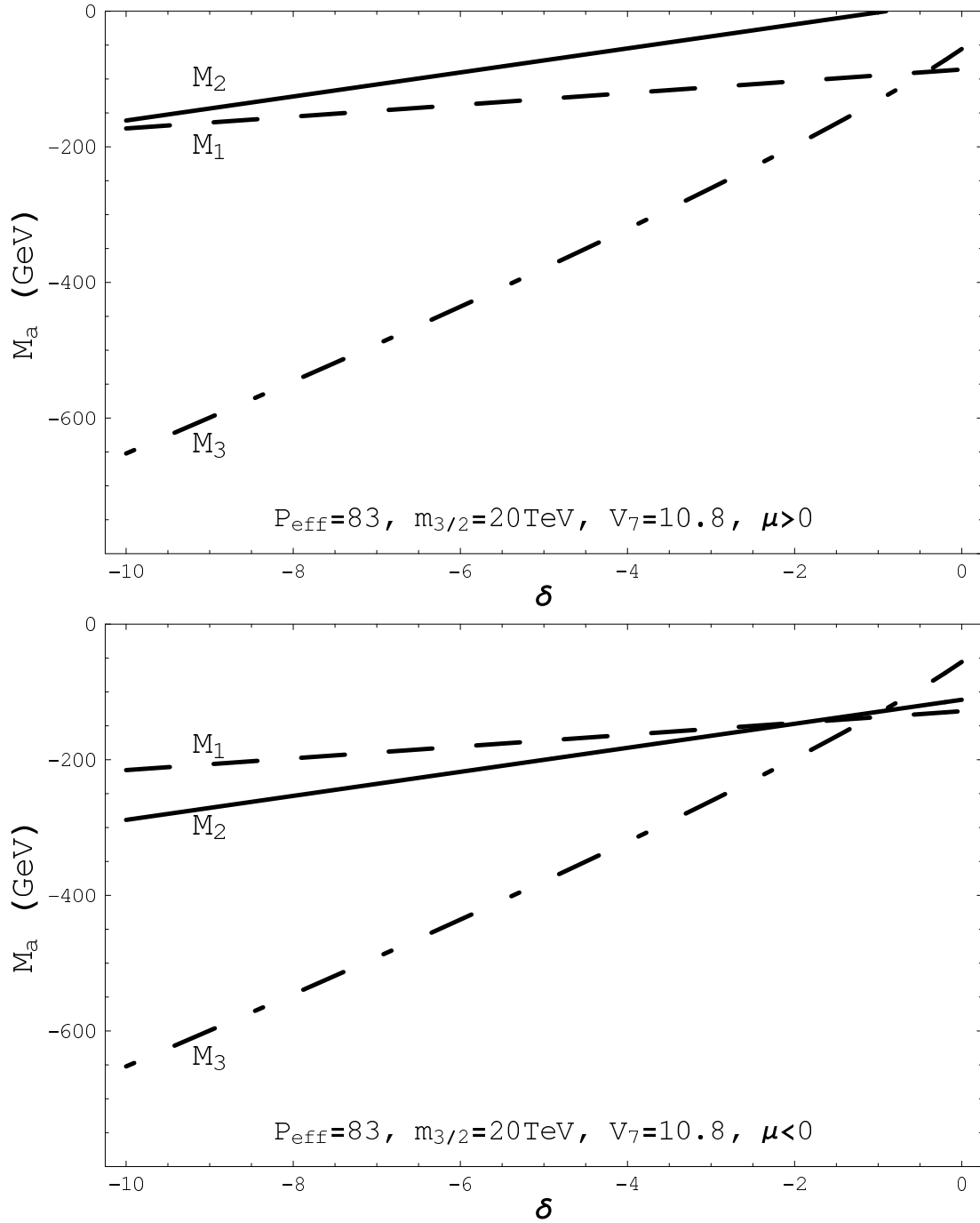


FIG. 2: Gaugino Masses at low scales (include correction (43)) as a function of δ for the case with cosmological constant tuned to zero ($P_{\text{eff}} = 83$), $Q - P = 3$, $V_7 = 10.8$, $C_2 = 1$ and $\alpha_{\text{unif}}^{-1} = 26.5$. TOP: Plot for positive μ . BOTTOM: Plot for negative μ .

Then, the hierarchy of gaugino masses mainly depends on δ - the threshold corrections to the unified gauge coupling. The dependence of gaugino masses on δ is shown in Figure 2. From the figures, we see that for $\mu > 0$, the wino tends to be the LSP ($|M_2| < |M_1|$) while for $\mu < 0$, the bino tends to be the LSP ($|M_1| < |M_2|$) for a large range in δ . Also, $|M_3|$ is significantly larger than $|M_1|, |M_2|$ for values of

$|\delta| \gtrsim 3$. As will be seen in section IV, $|\delta| \gtrsim 3$ is favored by precision gauge unification.

B. Electroweak Symmetry Breaking

Since the scalars in the G_2 -MSSM are generically very heavy, there are large logarithmic corrections to the Higgs potential. To analyze EWSB in such a theory, it is better to work in the low energy effective field theory in which all heavy fields are decoupled. Then, the large logarithmic corrections are automatically resummed when the Higgs parameters are RG evolved to the low scale. After decoupling all the heavy fields, the low energy effective theory is simply the standard model plus light gauginos. It is well known that in order to have electroweak symmetry breaking, there must exist a light Higgs doublet below the decoupling scale with negative mass parameter. It was pointed out in [24] that generically it is very hard to get EWSB predominantly from radiative effects below the decoupling scale, more so if the decoupling scale is not too high (as in our case). Therefore, if electroweak symmetry breaking happens in the effective theory at low scale, it should also happen in the MSSM theory at the decoupling scale. This means that we only need to check the existence of EWSB at the decoupling scale in the MSSM framework.

In order to do that, we have to first diagonalize the mass matrix of (h_u, h_d^*) :

$$\begin{pmatrix} m_{H_u}^2 + \mu^2 & -b \\ -b & m_{H_d}^2 + \mu^2 \end{pmatrix} \quad (45)$$

The eigenvalues are

$$\zeta_{1,2} = \frac{1}{2} \left[(m_u^2 + m_d^2) \pm \sqrt{(m_u^2 - m_d^2)^2 + 4b^2} \right], \quad (46)$$

where $m_u^2 = m_{H_u}^2 + \mu^2$ and $m_d^2 = m_{H_d}^2 + \mu^2$. The light Higgs doublet h is a superposition of h_u and h_d

$$h = \sin \beta h_u + \cos \beta h_d^* \quad (47)$$

where β is determined by the diagonalization of the matrix. In a complete high scale theory, the mass matrix Eq.(45) is completely determined by the high scale boundary condition. The existence of EWSB depends on whether there is one negative eigenvalue. As explained earlier, in realistic G_2 theory vacua, the effective μ and $B\mu$ terms in the low-energy lagrangian arise from the non-zero Higgs bilinear coupling Z . If the μ term in the original superpotential is forbidden by some discrete symmetry as in [13], then they are given by (see section IID):

$$\mu(M_{\text{unif}}) \approx z_{\text{eff}} m_{3/2}, \quad B\mu(M_{\text{unif}}) \approx 2z_{\text{eff}} m_{3/2}^2 \quad (48)$$

Using the above relation and RG evolving them to the decoupling scale, one can obtain a Higgs mass matrix which is parameterized by z_{eff} . One then finds that for $z_{\text{eff}} < z_{\text{eff}}^* \sim \mathcal{O}(1)$ there will always be a negative eigenvalue and so electroweak symmetry is broken. This condition for the existence of EWSB is naturally satisfied if $z_{\text{eff}} \sim \mathcal{O}(1)$. Since all elements in the mass matrix (45) are $\mathcal{O}(m_{3/2})$, the mixing coefficients ($\sin \beta$ and $\cos \beta$) of h_u and h_d^* are of the same order. Thus, $\tan \beta$ is naturally predicted to be of $\mathcal{O}(1)$. This is in contrast to usual approaches to high-scale model-building where μ and $B\mu$ are

completely unknown from theory and are only determined after fixing M_Z and choosing $\tan(\beta)$. In the G_2 -MSSM, $\tan \beta$ is not a free parameter and is determined by the relation between μ and $B\mu$ predicted from the theory.

Since all the elements in the mass matrix (45) are $\mathcal{O}(m_{3/2})$, the Higgs mass eigenvalues should also be $m_{3/2}$ which is around 100 times larger than the EW scale. This implies a fine-tuning if there is no magic cancellation. So, even though the existence of EWSB is generic, getting the correct Z mass is not. As will be discussed below, the requirement of obtaining the correct Z -boson mass fixes the precise value of z_{eff} . This requires a fine-tuning of z_{eff} .

In the following we describe the precise procedure used for obtaining EWSB with the correct Z mass. The heavy scalars are decoupled at M_s and the couplings of the low energy effective theory (consisting of the SM particles and the MSSM gauginos) are matched with those of the complete MSSM. Most importantly, the matching condition for the quartic coupling of the Higgs is given by:

$$\lambda(m_s) = \frac{\frac{3}{5}g_1^2 + g_2^2}{8} \cos^2 2\beta \quad (49)$$

It turns out that at energies below the decoupling scale M_s , the *one-loop* RG evolution of m (the SM Higgs mass parameter), λ and the Yukawa couplings is the same as that of the Standard Model:

$$\begin{aligned} 16\pi^2 \frac{d\lambda}{dt} &= 24\lambda^2 - 6y_t^4 + 12\lambda y_t^2 + \frac{27}{200}g_1^4 + \frac{9}{20}g_1^2 g_2^2 + \frac{9}{8}g_2^4 - \frac{9}{5}\lambda g_1^2 - 9\lambda g_2^2 \\ 16\pi^2 \frac{dm^2}{dt} &= m^2(6\lambda + 6y_t^2 - \frac{9}{10}g_1^2 - \frac{9}{2}g_2^2) \\ 16\pi^2 \frac{dy_t}{dt} &= y_t \left[\left(\frac{9}{2}y_t^2 + \frac{3}{2}y_b^2 + y_\tau^2 \right) - \left(\frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2 \right) \right] \\ 16\pi^2 \frac{dy_b}{dt} &= y_t \left[\left(\frac{3}{2}y_t^2 + \frac{9}{2}y_b^2 + y_\tau^2 \right) - \left(\frac{1}{4}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2 \right) \right] \\ 16\pi^2 \frac{dy_\tau}{dt} &= y_\tau \left[\left(3y_t^2 + 3y_b^2 + \frac{5}{2}y_\tau^2 \right) - \left(\frac{9}{4}g_1^2 + \frac{9}{4}g_2^2 \right) \right] \end{aligned} \quad (50)$$

It is important to mention that the above equations are different from the corresponding ones for split-supersymmetry because Higgsinos in the G_2 -MSSM are also very heavy (of $\mathcal{O}(m_{3/2})$) unlike that in split-supersymmetry [25]. Therefore, for the G_2 -MSSM, the Higgsinos also decouple below M_s forbidding additional terms which appear in the one-loop RG equations for split-supersymmetry. From above, the quartic coupling λ will get a large correction from RG evolution because of the large top Yukawa coupling. The gaugino masses M_a on the other hand will only receive one-loop corrections from gauge boson exchange. The corresponding RGE equations at one-loop are as follows:

$$16\pi^2 \frac{dM_1}{dt} = 0 \quad (51)$$

$$16\pi^2 \frac{dM_2}{dt} = -12b_2^2 M_2 \quad (52)$$

$$16\pi^2 \frac{dM_3}{dt} = -18b_3^2 M_3 \quad (53)$$

Given the boundary conditions for soft parameters for realistic M theory vacua as in section II, one finds that EWSB occurs if z_{eff} is of $\mathcal{O}(1)$. But the generic value of M_Z is around $m_{3/2}$, which can be seen from the fact that all the Higgs parameters are of the order of $m_{3/2}$. In order to get $M_Z = 91$ GeV, one has

to tune z_{eff} so that μ and $B\mu$ take values such that the lightest Higgs mass parameter comes out to be around M_{EW} . This fine-tuning is a manifestation of the *little hierarchy problem* - an unexplained hierarchy between the electroweak (SM-like Higgs) and superpartner (scalar) scales. Our current understanding of the theory does not yet allow us to explain the little hierarchy problem by a dynamical mechanism.

In the low energy effective theory the ratio of the Higgs mass to the Z mass turns out to be quite robust¹¹. The ratio is given by:

$$\frac{m_h}{m_Z} = \frac{2\sqrt{\lambda}(M_{EW})}{(\frac{3}{5}g_1^2 + g_2^2)(M_{EW})} \quad (54)$$

λ is determined by the gauge couplings, Yukawa couplings and $\tan\beta$. One has to use the boundary condition for λ at M_s as in (49) and then RG evolve it to the electroweak scale using the first equation in (50). The gauge couplings at the electroweak scale can also be determined by their RGEs. Thus, one can obtain the ratio m_h/m_Z as a function of $m_{3/2}$ as shown in Figure 3. It is worth noting that this ratio only mildly depends on $m_{3/2}$. If M_Z is tuned to its experimental value, we can use λ obtained from

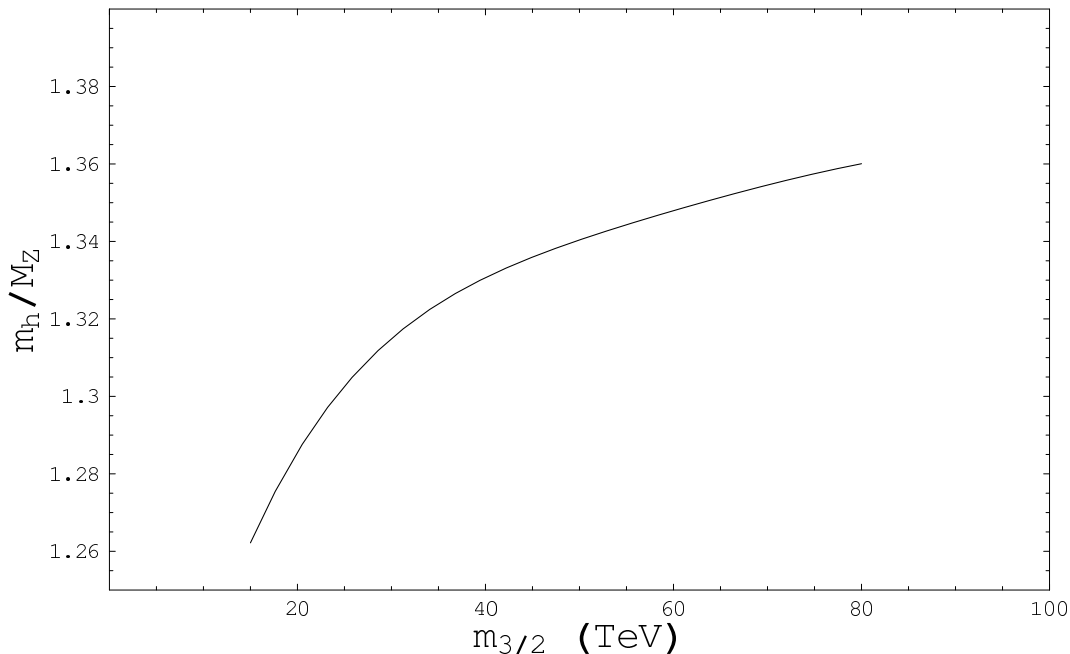


FIG. 3: Plot of the ratio of the Higgs mass and the Z mass as a function of the gravitino mass.

the RG equation or from Figure 3 to predict the Higgs boson mass for any given value of $m_{3/2}$. Once one finds the Higgs VEV v , the Higgs mass is simply $m_h^2 = 2\lambda v^2$ just as in the Standard Model since all heavy scalars and Higgsinos have already been decoupled. The Higgs boson mass thus computed turns out to be of $\mathcal{O}(120)$ GeV for a range of interesting values of $m_{3/2}$ as it only mildly depends on it. Since all susy-breaking large logarithms have already been taken into account in the ‘Match and Run’ procedure, only some finite term contributions could have been missed in this analysis. One could take those effects into account as well in a more detailed analysis, but that would not change the Higgs mass significantly.

¹¹ even when one does not tune z_{eff} to obtain the correct Z mass as explained in the previous paragraph.

The origin of the above value of the Higgs mass can be understood as follows. If one did not decouple the scalars and Higgsinos at M_s , then the Higgs mass receives very large radiative corrections, making the Higgs mass heavy as required. However, because of the hierarchy between the scalars and gauginos, these radiative corrections are hard to compute in a controlled manner. In the spirit of effective field theory therefore, it makes sense to integrate out the scalars and Higgsinos at M_s . In this picture, all the radiative corrections to the Higgs mass can be incorporated in the running of the quartic coupling λ . λ gets renormalized from M_s to M_{EW} giving rise to a heavy Higgs.

IV. GAUGE COUPLING UNIFICATION

In section IB, it was mentioned that in many cases the strong coupling limit of $E_8 \times E_8$ heterotic string theory compactifications on a Calabi-Yau threefold Z is the same as M theory compactifications on a singular G_2 -holonomy manifold X . Since a GUT-like spectrum is natural in weakly coupled heterotic compactifications, a GUT-like spectrum (breaking down to the MSSM by Wilson-lines) was assumed for G_2 compactifications in our study as well. At a theoretical level, because of an underlying GUT structure, the MSSM gauge couplings are unified at the compactification scale M_{KK} for both heterotic and G_2 compactifications. However, when one tries to impose constraints from the extrapolated values of observed gauge couplings, interesting differences arise between weakly coupled heterotic and G_2 compactifications. Here, we will first explain the difference between weakly coupled heterotic compactifications and G_2 compactifications regarding gauge unification and then discuss the procedure used in our analysis to obtain sets of parameters compatible with *precise* gauge unification.

In weakly coupled heterotic string compactifications, there is a relation between the Newton's constant G_N , the unified gauge coupling α_{unif} , the string coupling e^ϕ and the volume of the internal manifold V_{CY} [26]. Assuming a more or less isotropic Calabi-Yau, one has $V_{CY} \sim M_{\text{unif}}^{-6}$ which gives:

$$G_N \approx \frac{\alpha_{\text{unif}}^{4/3}}{4M_{\text{unif}}^2} \left(\frac{16\pi}{e^{2\phi}} \right)^{1/3} > \frac{\alpha_{\text{unif}}^{4/3}}{4M_{\text{unif}}^2} \quad (55)$$

since the string coupling is weak by assumption ($e^{2\phi} < 1$). Substituting the values of α_{unif} and M_{unif} obtained by extrapolating the observed gauge couplings in the MSSM, the prediction for G_N turns out to be too large compared to the observed value. Various proposals have been put forward for dealing with this problem within the perturbative heterotic setup, but none of them are obviously compelling.

In G_2 compactifications however, one finds a different relation among the same quantities¹² after doing a similar analysis [7]:

$$G_N \approx \frac{\alpha_{\text{unif}}^2}{32\pi^2 M_{\text{unif}}^2} \left(\frac{1}{a} \right); \quad a \equiv \frac{V_7}{V_{\hat{Q}_{vis}}^{7/3}} \quad (56)$$

Here, “ a ” is the dimensionless ratio between the volume of the G_2 manifold V_7 and the volume of the three-manifold \hat{Q}_{vis} on which the visible sector MSSM gauge group is supported. If one does a more

¹² in M theory however, there is no string coupling.

careful analysis and takes into account the threshold corrections to the unified gauge coupling from Kaluza-Klein (KK) modes, one obtains [7]:

$$G_N = \frac{\alpha_{\text{unif}}^2}{32\pi^2 M_{\text{unif}}^2} \left(\frac{(L(\hat{Q}_{vis}))^{2/3}}{a} \right) \quad (57)$$

where $L(\hat{Q}_{vis})$ is the contribution from the threshold correction. Substituting the values of α_{unif} and M_{unif} obtained by extrapolating the observed gauge couplings in the MSSM and the value of G_N , one finds:

$$\left(\frac{(L(\hat{Q}_{vis}))^{2/3}}{a} \right) \approx 15 \quad (58)$$

In all examples where duality with heterotic string theory or Type IIA string theory is used to deduce the existence of the G_2 manifold X , “ a ” is expected to be much less than unity [7]. For G_2 -MSSM vacua, with natural values of the microscopic parameters one obtains $V_7 = 10 - 100$ [1] while $V_{\hat{Q}_{vis}} \sim \alpha_{\text{unif}}^{-1} \sim \mathcal{O}(25)^{13}$. Thus, $a \equiv \frac{V_7}{V_{\hat{Q}_{vis}}^{7/3}} \ll 1$ is also naturally satisfied for G_2 -MSSM vacua. In addition, by expressing V_7 and $V_{\hat{Q}_{vis}}$ in terms of M_{11} and M_{unif} respectively, $a < 1$ implies $M_{\text{unif}} < M_{11}$ which means that the unification scale constraint stated in section II can be naturally satisfied. Since $a \ll 1$, from (58) one requires:

$$(L(\hat{Q}_{vis}))^{2/3} \ll 15 \quad (59)$$

The quantity $L(\hat{Q}_{vis})^{2/3}$ depends on certain topological invariants of the three-manifold \hat{Q}_{vis} and can be computed for special choices of \hat{Q}_{vis} . For one such choice - $\hat{Q}_{vis} = \mathbf{S}^3/Z_q$; $q \in \mathbf{Z}$, $(L(\hat{Q}_{vis}))^{2/3}$ has been computed [7]. It depends on two integers ω, q such that 5ω is not a multiple of q .¹⁴ Figure 4 shows the variation of $L^{2/3}$ for $\hat{Q}_{vis} = \mathbf{S}^3/Z_q$ as a function of q for different choices of $5\omega \bmod q$. One sees that $L^{2/3} \ll 15$ can be obtained in a natural manner for a large range of q . For other choices of \hat{Q}_{vis} , it is reasonable to expect a similar result. To summarize therefore, G_2 -MSSM vacua are naturally compatible with gauge unification in general and the “unification scale constraint” mentioned in section II in particular.

The value of the unified gauge coupling α_{unif} is also affected by the threshold corrections. The tree-level unified gauge coupling of the visible sector at the compactification scale is the volume of the visible sector three-manifold \hat{Q}_{vis} :

$$\alpha_{vis}^{-1} \equiv \text{Im}(f_{vis}^0) = V_{\hat{Q}_{vis}} = \sum_{i=1}^N N_i^{vis} s_i, \quad (60)$$

After taking into account the threshold corrections (at one-loop), one has:

$$\alpha_{\text{unif}}^{-1} = \alpha_{vis}^{-1} + \delta \quad (61)$$

For the one-loop result to be reliable, δ should be small compared to α_{vis}^{-1} . Since for the MSSM $\alpha_{unif}^{-1} \sim 25$ one requires $\alpha_{vis}^{-1} \sim \mathcal{O}(25)$ as well. The conditions under which the microscopic parameters can give rise

¹³ The unified coupling constraint will be discussed in Appendix B.

¹⁴ It is assumed that an $SU(5)$ GUT group is broken to the MSSM by Wilson lines.

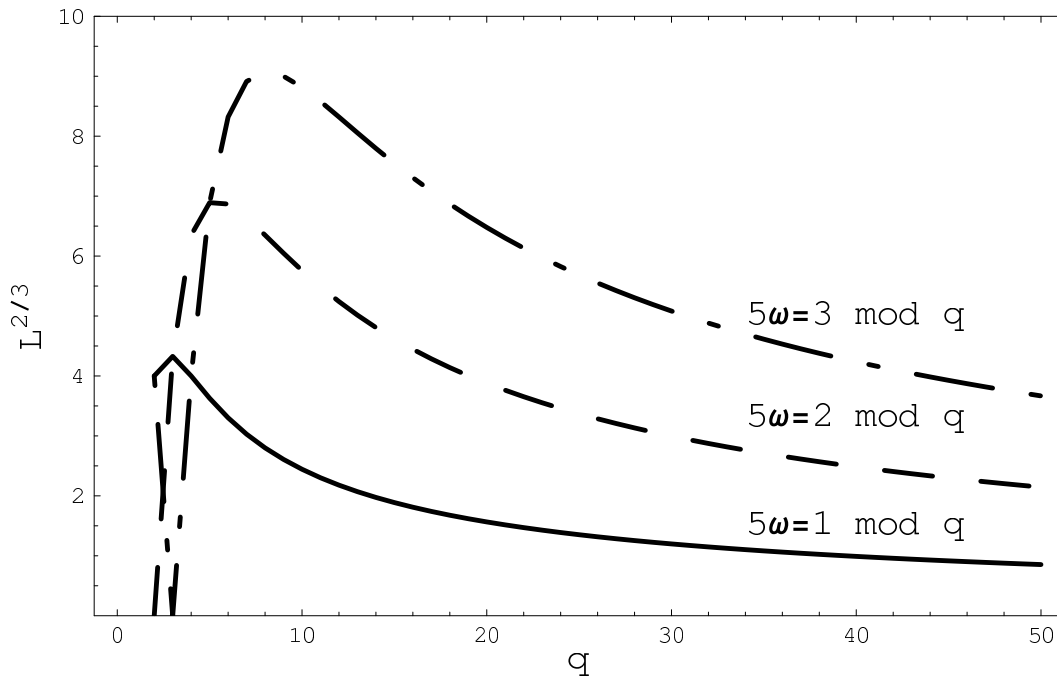


FIG. 4: Plot of $L^{2/3}$ as a function of integer q from 2 to 50. Different curves correspond to different choices of ω as marked in the plot.

to the above value of α_{vis}^{-1} is discussed in Appendix B. The threshold correction δ can be computed from the topological invariants of the three-manifold \hat{Q}_{vis} , so it also depends on integers characterizing the topology of \hat{Q}_{vis} . However, it is in general regularization dependent in contrast to expression (57) for the unification scale [7]. Therefore, in our analysis we will assume δ to be a free parameter varying in a reasonable range such that one-loop results are still reliable.

A. Precision Gauge Unification

Having convinced ourselves that G_2 -MSSM vacua are naturally compatible with gauge coupling unification, we will now examine the issue of precision gauge coupling unification in the G_2 -MSSM in the sense that we would like to find sets of microscopic parameters which give rise to precise gauge unification. There are two aspects to this issue: a) Is there a unification of gauge couplings at a high scale $\mathcal{O}((1-3) \times 10^{16})$ GeV by continuing the gauge couplings up in energy from the laboratory scale including all the low-scale thresholds, and b) Whether the unified gauge coupling and the unification scale obtained are consistent with the theoretical prediction in terms of “microscopic” parameters. As we will see, it turns out that the G_2 -MSSM is compatible with precision gauge coupling unification in the sense that there exists a relatively large set of reasonable microscopic parameters which gives rise to precise gauge coupling unification.

Before going into details, it is important to notice the following general fact – gaugino masses at the unification scale and hence the low scale depend on the value of α_{unif}^{-1} . However, the value of α_{unif}^{-1} itself depends on corrections to the gauge couplings from superpartner thresholds at low scale. This means

that there is a feedback between the spectrum of superpartner masses and the value of α_{unif}^{-1} . Therefore, one has to remember to take into account the effects of this feedback in general. Since the squarks and

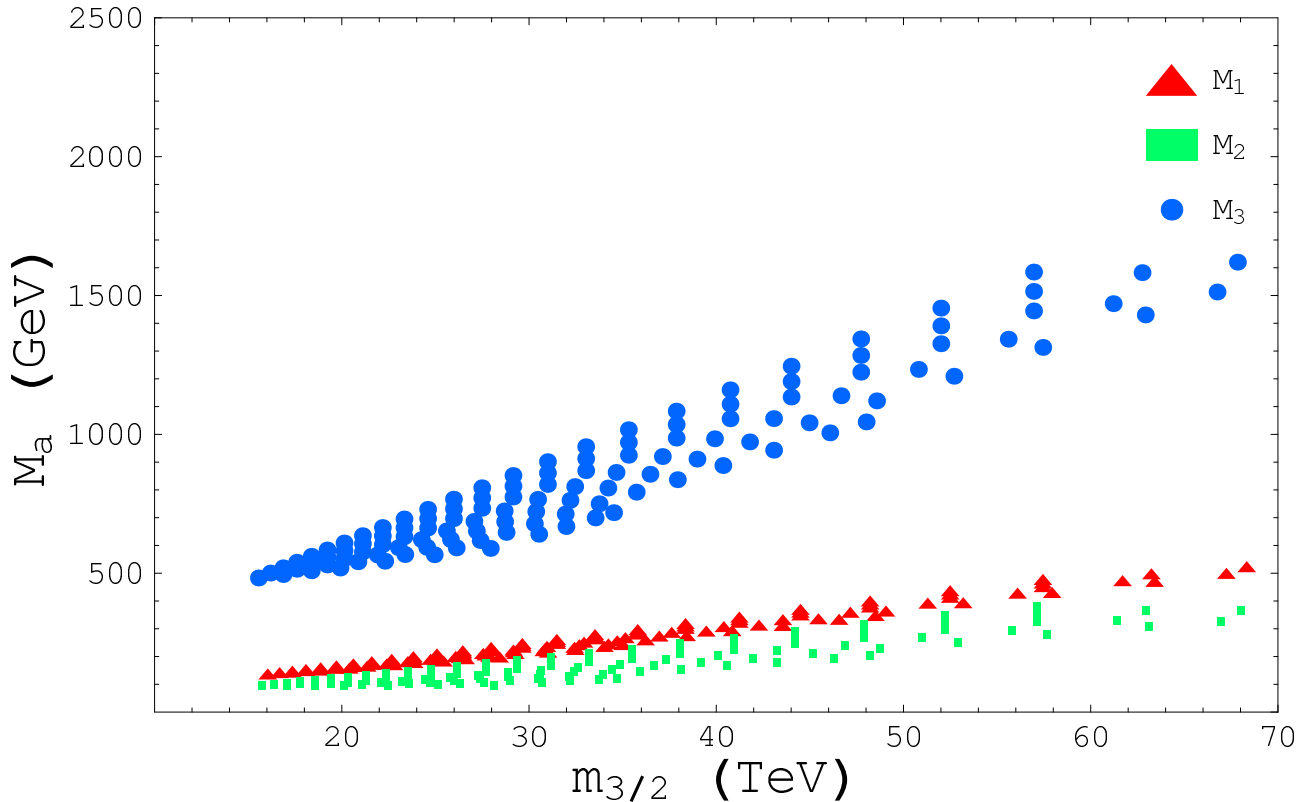


FIG. 5: Gaugino Mass spectra vs $m_{3/2}$ compatible with gauge unification for $P_{\text{eff}} = 83$, $C_2 = 5$ and $\mu > 0$. The red, green and blue lines correspond to gaugino mass M_1 , M_2 and M_3 respectively.

sleptons come in complete GUT multiplets, they do not affect gauge unification. The Higgs doublets, Higgsinos and gauginos do affect gauge unification since they do not form complete GUT multiplets. Since the Higgsino mass (μ) in these vacua are heavy ($\mathcal{O}(m_{3/2})$) and is robustly determined by the gravitino mass once the EWSB breaking constraint is imposed, for a fixed gravitino mass gauge coupling unification will mostly depend on the light gaugino masses, the gaugino mass ratio $|M_3|/|M_2|$ in particular because it contributes the most to the threshold corrections to the gauge couplings. We find that in order to have precise unification, this ratio has to be greater than around 3 – 4. This sensitivity to $|M_3|/|M_2|$ is much greater here than in split-SUSY where both the Higgsinos and gauginos are light. Finally, if there are particles in the GUT multiplet in addition to the MSSM (the Higgs triplets in the $SU(5)$ case for example) that are lighter than M_{unif} , then one should also take their threshold contributions to the gauge couplings into account. However one finds that their threshold contribution causes α_3^{-1} and α_1^{-1} to move *away* from each other. Therefore, the requirement of precision gauge unification forces us to assume that such particles (like the triplets) are at least as heavy as the unification scale. It seems possible to arrange that in many models.

Based on the above arguments, we performed a complete scan over the parameter space on which the gaugino masses depend – $\{\delta, V_7, C_2, \alpha_{vis}^{-1}\}$, assuming $Q - P = 3$, $P_{\text{eff}} = 83$. As negative δ is necessary to obtain the right unification scale, we take a range $-10 \leq \delta \leq 0$. C_2 is taken to be $\mathcal{O}(1)$. $\alpha_{vis}^{-1} \equiv \text{Im}(f^0)$

is taken to be of $\mathcal{O}(25)$. The lower and upper bounds on V_7 are given by ¹⁵:

$$\begin{aligned} V_7^{min} &= 1; \text{ weak supergravity constraint (Appendix B)} \\ V_7^{max} &= V_{\hat{Q}_{vis}}^{7/3} \approx (\alpha_{unif}^{-1} - \delta)^{7/3}; \text{ corresponding to } a \equiv \frac{V_7}{V_{\hat{Q}_{vis}}^{7/3}} = 1 \end{aligned} \quad (62)$$

In addition, we consider a gravitino mass below 100 TeV so that the spectrum is light enough to be potentially be seen at the LHC, as well as satisfy all our constraints. The analysis was done for two different cases $\mu > 0$ and $\mu < 0$. As explained in section III A, M_1 and M_2 at low scale get contributions from the Higgs-Higgsino loop (see eqn. (43)) which depends on the sign of μ . This means that for a given $m_{3/2}$ which is consistent with all mass bounds on the spectrum, $|M_3|/|M_2|$ for $\mu > 0$ is greater than that for $\mu < 0$. So, it turns out that $\mu < 0$ is excluded by gauge coupling unification while $\mu > 0$ is allowed. The gaugino mass spectra for $\mu > 0$ compatible with precision gauge unification and all bounds on superpartner masses are shown in Figure 5. The most stringent bound among superpartner mass bounds is that of the lightest chargino from LEP II. For the bino LSP case, the bound is $M_{\tilde{C}_1} \geq 104$ GeV. However for a wino LSP, which turns out to be relevant for us, the bound depends on the mass splitting $\Delta M \equiv M_{\tilde{\chi}_1^\pm} - M_{\tilde{\chi}_1^0}$; for simplicity we take $M_{\tilde{C}_1} \geq 80$ GeV.

The procedural details used are as follows. For a choice of δ , C_2 , V_7 and α_{unif}^{-1} in the above range as well as for a set of initial values of Yukawa couplings and z_{eff} at $M_{\text{unif}} \sim \mathcal{O}(10^{16})$ GeV, the MSSM spectrum was computed at low scales using the analysis in sections II C and III A. The experimental values of the gauge couplings (both their max. and min. values taking the uncertainty into account) were then RG evolved backwards to the high scale using two-loop RGEs which depend on the superpartner thresholds. The unified gauge coupling and the unification scale were determined by the requirement $\alpha_1(M_{\text{unif}}) = \alpha_2(M_{\text{unif}})$. The Yukawa couplings were evolved to the high scale at the same time. The original parameters were scanned within their respective ranges and only those values for which the initial assumed α_{unif} was equal to the value of the computed α_{unif} up to experimental uncertainties, were recorded. M_Z was checked to be approximately 91 GeV. The condition for gauge coupling unification, i.e. $\alpha_{3,min}(M_{\text{unif}}) < \alpha_{1,2}(M_{\text{unif}}) < \alpha_{3,max}(M_{\text{unif}})$, was checked and only sets of parameters which satisfied the above condition as well as other constraints on superpartner masses, were recorded. In the above condition, $\alpha_{3,min}(M_{\text{unif}})$ and $\alpha_{3,max}(M_{\text{unif}})$ are the lower and upper values of α_3 at the unification scale, determined by RG evolving the low scale experimental value of α_3 taking the uncertainties into account. The low scale gaugino mass spectra consistent with precise gauge coupling unification are plotted in Figure 5. As explained earlier, this is only possible for $\mu > 0$. One sees from Figure 5 that only discrete values of gaugino masses are possible since it is not possible to satisfy precision gauge unification constraints for continuous sets of parameters.

¹⁵ The upper bound is determined from the fact that in both heterotic and type IIA duals of these vacua, the parameter $a \equiv \frac{V_7}{V_Q^{7/3}}$ is always less than unity [7].

V. WHAT IS THE LSP?

From Figure 5, the lightest supersymmetric particle (assuming R -parity conservation) turns out to be predominantly wino-like. The Higgsinos are of $\mathcal{O}(m_{3/2})$ and are much heavier than the gauginos. Here, as usual we have assumed that $Q - P = 3$ and $P_{\text{eff}} = 83$.

It is worthwhile to compare and contrast the results obtained for the G_2 -MSSM for the nature of the LSP with those for the Type IIB vacua corresponding to the “mirage mediation” framework mentioned in the introduction. There one always gets bino LSPs. In mirage mediation, the gaugino mass contribution is dominated by the tree-level and conformal anomaly contribution (the first term in (29)) which are of the same order [18]. The second and third term in (29) are negligible because of the assumption of sequestering. On the other hand, for the G_2 -MSSM, the Konishi anomaly contribution coming from the second and third term in (29) is also important as one does not expect sequestering in general. The second important difference is that the μ parameter is very large for the G_2 -MSSM (of $\mathcal{O}(10)$ TeV) compared to that for mirage mediation. This implies that the finite contribution to M_1 and M_2 from (43) in the G_2 -MSSM is quite important in contrast to that in mirage mediation. Also, as seen from the bottom plot in Figure (2), $\mu < 0$ gives rise to bino LSPs; however only G_2 -MSSM vacua with $\mu > 0$ should be considered since those with $\mu < 0$ are disfavored by precision gauge coupling unification as explained in section IV. Therefore, due to all the above reasons, the nature of the LSP obtained for the G_2 -MSSM is different from that for mirage mediation.

A. Dark Matter Relic Abundance and Cosmological Evolution of Moduli

It is well known that wino LSPs annihilate quite efficiently compared to bino LSPs due to the larger $SU(2)$ gauge coupling g_2 . Therefore, for wino masses of $\mathcal{O}(100\text{-}500)$ GeV, as is natural for the G_2 -MSSM vacua, the thermal contribution to the relic density of these wino LSPs is much smaller ($\sim 0.01\text{--}0.1$ times) than the observed upper bound on the relic density. This implies that if (wino) LSPs constitute most of the Dark-Matter(DM) in the universe, they must be produced non-thermally. The issue of non-thermal production mechanisms of these LSPs is intricately linked to the cosmological evolution of moduli after inflation. As it turns out, during the course of their evolution moduli decay to LSPs giving rise to an appreciable non-thermal contribution which could give rise to a wino LSP relic density of approximately the right amount. This will be reported in detail in a future study [27]. Here we will just outline the salient features of our analysis of cosmological moduli evolution.

It is well known that light gauge-singlet scalar fields such as moduli and meson fields couple very weakly (only gravitationally) to the visible sector causing them to decay at late times which in turn could spoil the successful predictions of Big-Bang Nucleosynthesis (BBN). This is a generic problem in string/ M theory compactifications and has to be addressed carefully. In G_2 compactifications, since all moduli are stabilized, the moduli mass matrix, as well as all couplings of the moduli to the visible sector can be explicitly computed in terms of the microscopic parameters. It turns out that there is a hierarchy of mass scales for the moduli with one modulus being much heavier than the others, the lighter ones being $\mathcal{O}(2m_{3/2})$. To set the stage for the analysis, it is reasonable to assume that enough inflation occurs at the end of which reheating gives rise to a radiation dominated universe with the moduli displaced from their minimum values. Then, one has to look at the evolution of these moduli carefully, taking

into account the hierarchy of scales involved. During the course of their evolution, the moduli will start coherent oscillations and then decay to visible particles ultimately leading to SM particles and LSPs. Since all the relevant moduli-matter couplings can be explicitly computed, this sequence of steps can be carried out reliably and the LSP relic density can be computed. It turns out that the wino LSP relic density for natural values of parameters is in the range 0.1-1. The cosmological evolution of moduli in G_2 -MSSM vacua is, in a sense, an explicit realization of the Randall-Moroi scenario [28] in which the moduli are quite heavy ($10 - 100$ TeV) and decay before BBN to wino-like LSPs. The detailed spectrum is however different and more detailed computations can be done for the G_2 -MSSM vacua.

VI. BENCHMARK SPECTRA AND CHARACTERISTIC FEATURES

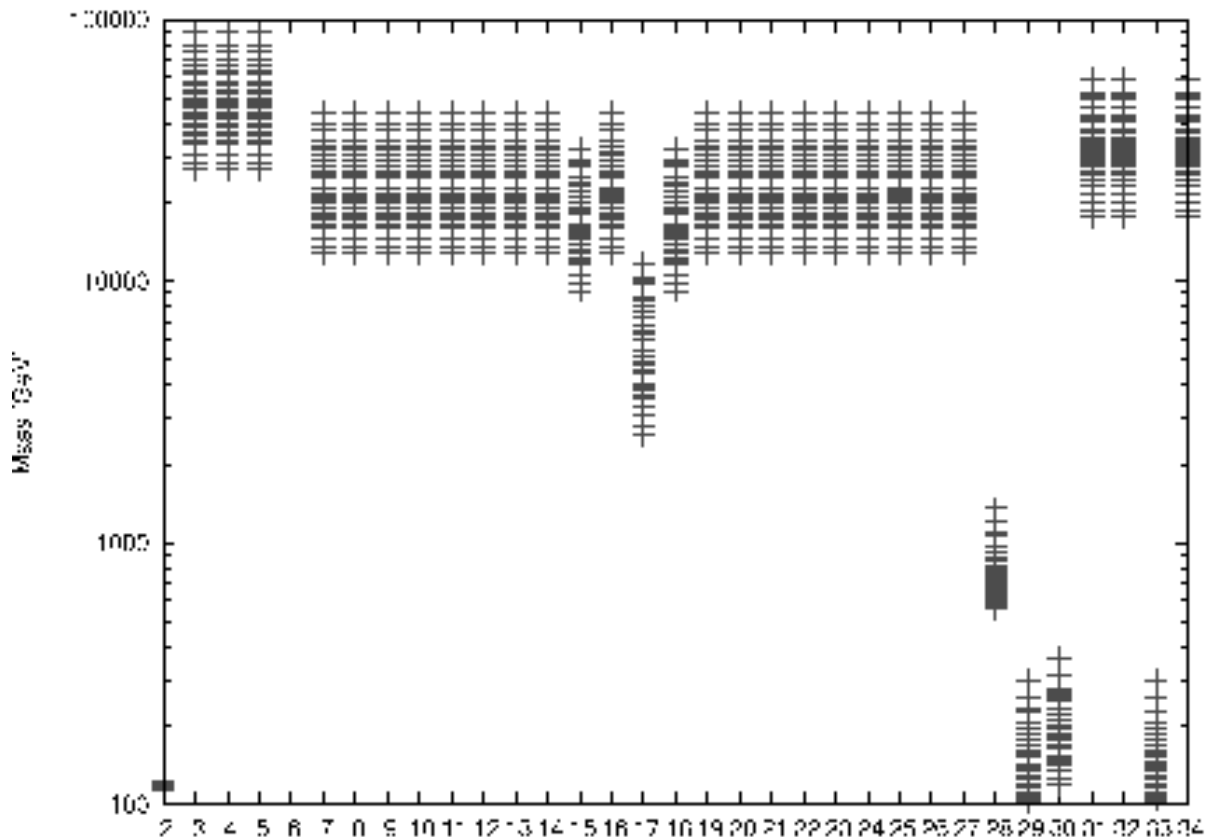


FIG. 6: Semi-log Plot showing variations in the spectra of G_2 -MSSM models. The columns correspond to the following masses. 2: Light Higgs (h). 3-5: Heavy Higgs/pseudo-scalar Higgs. 7-14: 1st/2nd Generation squarks. 15-18: 3rd Generation squarks. 19-27: Sleptons. 28: gluino. 29: \tilde{N}_1 . 30: \tilde{N}_2 . 31-32: \tilde{N}_{3-4} . 33-34: \tilde{C}_{1-2} .

In this section, we take a brief look at the pattern of spectra obtained in the G_2 -MSSM and point out their main features. A detailed study of the collider phenomenology of the G_2 -MSSM will be reported in [29]. Table I shows the ‘microscopic’ input parameters and corresponding low-scale spectra for four benchmark G_2 -MSSM models while Figure 6 displays the physical masses for many G_2 -MSSM models. As discussed in section III, the G_2 -MSSM spectrum is characterized by heavy multi-TeV scalars and

parameter	Point 1	Point 2	Point 3	Point 4
δ	-4	-6	-8	-10
$m_{3/2}$	67558	35252	34295	17091
V_7	14	21.6	22	35
α_{unif}^{-1}	26.7	26.4	26.5	26.0
Z_{eff}	1.58	1.65	1.65	1.77
$\tan\beta$	1.44	1.45	1.45	1.45
μ	87013	45572	44164	22309
$m_{\tilde{g}}$	994.7	732.5	900.4	573.5
$m_{\tilde{\chi}_1^0}$	116.6	110.9	173.1	107.1
$m_{\tilde{\chi}_2^0}$	390.0	228.3	253.5	137.1
$m_{\tilde{\chi}_1^\pm}$	116.7	111.0	173.2	107.3
$m_{\tilde{u}_L}$	67600	35254	34298	17094
$m_{\tilde{u}_R}$	67559	35264	34298	17093
$m_{\tilde{t}_1}$	18848	9010	8700	3850
$m_{\tilde{t}_2}$	49554	25707	24998	12378
$m_{\tilde{b}_1}$	49554	25707	24998	12378
$m_{\tilde{b}_2}$	67497	35220	34265	17076
$m_{\tilde{e}_L}$	67558	35253	34296	17091
$m_{\tilde{e}_R}$	67559	35253	34296	17091
$m_{\tilde{\tau}_1}$	67527	35237	34280	17084
$m_{\tilde{\tau}_2}$	67543	35245	34288	17088
m_h	123.6	120.8	120.3	118.1
m_A	134083	70053	68031	34107
A_t	14267	6208	6024	2379
A_b	3114	1637	1604	805
A_τ	1935	972	965.5	468.7

TABLE I: “Microscopic” parameters and low scale spectra for four benchmark G_2 -MSSM models. All masses are in GeV. The top mass is taken to be 174.3 GeV in our calculation. For all the above points, $Q - P = 3$ and $P_{\text{eff}} = 83$ are taken as discussed in the text. The gravitino mass depends mainly on the combination $C_2 V_7^{-3/2}$ as in Eq. (15). So the spectra are largely determined by two parameters δ and $m_{3/2}$. All the above spectra are consistent with current observations. Scalar masses are lighter for benchmark 4, so flavor changing effects need to be explicitly checked later.

Higgsinos, sub-TeV gauginos, and an SM-like Higgs field¹⁶. Thus, the arrangement of the sub-TeV fields crucially determines the pattern of observable signatures at the LHC.

G_2 -MSSM models with $Q - P = 3$, $P_{\text{eff}} = 83$ and which are consistent with precision gauge coupling unification give rise to wino LSPs. For this case, the following hierarchy between the sub-TeV particles is observed:

$$m_{\tilde{g}} > m_{\tilde{N}_2} > m_{\tilde{C}_1} > m_{\tilde{N}_1} \quad (63)$$

¹⁶ The parameter $\tan(\beta)$ is of $\mathcal{O}(1)$.

The \tilde{N}_1 is nearly degenerate with the \tilde{C}_1 ($m_{\tilde{C}_1} - m_{\tilde{N}_1} \lesssim 200$ MeV). Variations of high-scale input parameters V_7 and δ simply shift the overall mass scale of the fields, but do not modify this hierarchy. This feature significantly constrains the possible decay modes observable at the LHC. For example, the dominant production modes are $\tilde{C}_1\tilde{C}_1$ and $\tilde{C}_1\tilde{N}_1$ followed by $\tilde{g}\tilde{g}$. The decay of the charginos to the LSP is characterized by soft pions. The charginos tend to decay inside the detector giving rise to short charged track stubs. A systematic study of these signatures in both ATLAS and CMS is required to properly estimate the discovery potential of these decays. The gluinos on the other hand decay dominantly via a three-body decay to $t\bar{t}\tilde{N}_2$ since the lightest stop is mostly right handed. This mode can give rise to signatures with many b-jets from multi-top production. Other decay modes, although suppressed, are also available. A study of these issues is underway and will be reported in [29].

The tree level production rate for $\tilde{C}_1 + \tilde{N}_2$ would vanish for pure bino \tilde{N}_2 so the size of this cross section is a measure of the wino part of the LSP. Similarly, the rate for production of $\tilde{C}_1 + \tilde{N}_1$ is about two times larger for a wino LSP than for a higgsino LSP and can thus help fix the LSP type. The signal for us is the single soft charged pion, which will not trigger, but can be triggered in association with an initial state photon or jet, and the chargino will pass a few layers of the vertex detectors so in principle these rates might be accessible.

Since all the Higgs bosons except the light one (h) will be in the TeV range, and undetectable, it is interesting to have ways to distinguish the light Higgs (h) from the SM one (h_{SM}). One way to do that in principle is to exploit the chargino loops in the modes $h \rightarrow \gamma\gamma$ and $h \rightarrow Z\gamma$, which will make the branching ratio different from the SM one. Accurate measurements would be needed, and effects of CP violation would have to be untangled to obtain definitive results.

Finally, we mention that the fit to EW precision observables apparently improves with light charginos and neutralinos [30].

VII. CONCLUSIONS AND FUTURE DIRECTIONS

In this paper, we have studied a framework arising from the low-energy limit of M theory which gives rise to vacua in which the moduli can be stabilized and a stable hierarchy between the electroweak and Planck scales can be generated. A well-motivated phenomenological model - the G_2 -MSSM, can be naturally defined within this framework and its properties can be studied in detail. The model arises by compactifying M theory on a seven dimensional singular manifold of G_2 holonomy. Strong gauge dynamics in the hidden sector simultaneously stabilizes all the moduli and breaks supersymmetry. With matter fields in at least one of the hidden sectors, there is a de Sitter minimum that is unique for a given choice of G_2 -manifold. We look for solutions which are within the supergravity regime, where the volume of the three-manifold supporting the hidden sector is large, the number of moduli is not constrained to be small, and the cosmological constant can be tuned to zero.

Then, remarkably, we find that all solutions have $m_{3/2}$ less than a few hundred TeV and the suppression of gaugino masses leads to TeV and sub TeV scale new physics. The requirement that the cosmological constant can be tuned to zero is non-trivial. This constraint strongly depends on the nature of the three-manifolds on which the hidden sectors are supported. At present this computation has been carried out only for a very special class of three-manifolds $\mathbf{S}^3/\mathbf{Z}_k$; $k \in \mathbf{Z}$, where it turns out that the above requirement can only be satisfied for large values of k , which may not be very natural. However,

as discussed in Appendix A, various other possibilities for three-manifolds exist which might help in satisfying the requirement more naturally.

Further, we find that tree level gaugino masses are suppressed compared to $m_{3/2}$ by a large factor, approximately 83. This occurs mainly because the largest supersymmetry breaking F -term is that from hidden sector meson fields, which do not contribute to the gaugino masses. No such suppression occurs generically for scalar masses or trilinears, which are therefore of order $m_{3/2}$. Since the Kahler potential is not sequestered, the scalar masses are expected to be of order $m_{3/2}$, with the gaugino masses much smaller. There is no theoretical lower limit on $m_{3/2}$, but the absence of charginos at LEP gives a lower limit on the gaugino masses M_1 and M_2 , and therefore a lower limit on $m_{3/2}$, which is of order 10-50 TeV. So, scalars are predicted to be that heavy.

We focus on solutions within the G_2 -MSSM which are consistent with precision gauge coupling unification. Then the LSP is wino-like. The G_2 -MSSM framework is broadly similar to other models with heavy scalars and light gauginos with wino LSPs [31], but there are differences in details. As is well known, the thermal relic density of wino LSPs is small compared to the observed one. However, many LSPs are generated from moduli decay, along with significant entropy. When the cosmological evolution of moduli is taken into account the resulting relic density is near the observed one. The moduli masses and lifetimes can be computed in detail in terms of the microscopic details, and we find a nice explicit realization of the Moroi-Randall [28] mechanism¹⁷. The predicted LHC signatures are characteristic and interesting. RG evolution down to the gluino mass \lesssim a TeV leads to the lightest squark being a mainly right handed stop, which itself decays mainly to the top and the second neutralino, which then decays to the W and the lightest chargino (wino). Thus, for pair produced gluinos, there are many events corresponding to a final state with four tops, large missing energy, and two charginos, dramatic signatures that are easily triggered on and distinguishable from background. We also initiated analysis of the CP violating phases of the theory, which are surprisingly simple.

There are many possibilities for future research. From a theoretical perspective, one of the most outstanding problems is to construct global examples of G_2 manifolds with the right structure of conical and orbifold singularities. This would require a major breakthrough from a mathematical point of view. However, a better understanding of the dualities from the heterotic and Type IIA string could also lead to important insights. Another important theoretical issue is to better understand the assumptions made about the Kähler potentials and study possible generalizations. Although we have checked that our main qualitative results about the suppression of gaugino masses do not depend on the detailed form of the Kähler potential K , further insights would be welcome.

From a phenomenological perspective, it would be extremely useful to study variants of the minimal proposal which could solve important phenomenological problems while still retaining the desirable features. A good feature of this framework is that important phenomenological questions such as inflation, generation of neutrino masses, explanation of Yukawa couplings and the origin of flavor¹⁸, the matter asymmetry, the strong CP problem, the little hierarchy problem, etc. can all be addressed within this framework.

¹⁷ In details, the G_2 -MSSM differs from the Moroi-Randall scenario, however.

¹⁸ This is intrinsically related to the issue of the Kähler potential mentioned above.

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APPENDIX A: COMPUTATION OF P_{eff}

As we have seen in section I A, a large P_{eff} is crucial for the validity of our solutions and the supergravity approximation. It also leads to the suppression of tree-level gaugino masses compared to the gravitino mass. Finally, in order to tune the cosmological constant, one requires $P_{\text{eff}} = 84$ (83 if one includes higher order corrections) for $Q - P = 3$. In this paper, until now we have just used P_{eff} as one of the parameters which could vary within a certain range. However, in an explicit microscopic construction of the hidden sector, it is computable from first principles. From the definition:

$$P_{\text{eff}} \equiv P \ln \left(\frac{C_1}{C_2} \right) \quad (\text{A1})$$

it is easy to see that for a large P_{eff} such as 83, a large splitting in the coefficients C_1 and C_2 is required. At tree level, these coefficients are simply determined by the cutoff scale of the effective gauge theory and are given by $(\Lambda_{\text{cutoff}}/M_P)^3$. This would not give a large P_{eff} . However, one has to take into account threshold corrections to the gauge couplings of the hidden sectors. To compute the threshold corrections, one has to specify the concrete setup of the hidden sector \hat{Q} , as well as the geometry of the three-manifold where the hidden sector lives. Generally the one-loop gauge couplings can be written as:

$$\frac{16\pi^2}{g^2(\mu)} = \frac{16\pi^2}{g_M^2} + b \log(\Lambda^2/\mu^2) + S, \quad (\text{A2})$$

where b is the one-loop beta function coefficient, and S is the one-loop threshold correction. For instance, the contribution from KK modes has the form [7]:

$$S = S' + 2N_c \log(\text{Vol}(\hat{Q})\Lambda_{\text{cutoff}}^3), \quad (\text{A3})$$

where $\text{Vol}(\hat{Q})$ is the volume of the hidden-sector three-manifold \hat{Q} and S' can be expressed in terms of certain topological invariants of \hat{Q} , known as the ‘‘Ray-Singer analytic torsion’’. Before we go to explicit examples, we would like to show the general requirement on the threshold corrections. Let us first denote the gauge kinetic function as $f = f_0 + \Delta f_1$ and $f_2 = f_0 + \Delta f_2$, where $\Delta f_{1,2}$ are the corresponding threshold correction. The superpotential from strong gauge dynamics can be written as:

$$W \sim \Lambda_{\text{cutoff}}^{3+\alpha} |\phi|^{-\alpha} P e^{-\frac{2\pi}{P}(f+\Delta f_1)} + \Lambda_{\text{cutoff}}^3 Q e^{-\frac{2\pi}{Q}(f+\Delta f_2)} \quad (\text{A4})$$

We can easily identify the coefficient $C_{1,2}$ as follows:

$$C_1 = \left(\frac{\Lambda_{\text{cutoff}}}{M_P} \right)^{3+\alpha} e^{-\frac{2\pi}{P}\Delta f_1} \quad (\text{A5})$$

$$C_2 = \left(\frac{\Lambda_{\text{cutoff}}}{M_P} \right)^3 e^{-\frac{2\pi}{P}\Delta f_2} \quad (\text{A6})$$

Since $\alpha = 2/P$ is small, we have

$$\frac{C_1}{C_2} \approx e^{-\frac{2\pi}{P}\Delta f_1 + \frac{2\pi}{Q}\Delta f_2}. \quad (\text{A7})$$

For the case $Q - P = 3$ and $P_{\text{eff}} = 84$, using Eq.(A1) and (A7) we have the estimate

$$\Delta f_1 - \Delta f_2 \sim 14. \quad (\text{A8})$$

In view of the fact that $f_0 \approx \frac{14}{3}Q = \mathcal{O}(50)$, the requirement Eq.(A8) is not completely unreasonable.

1. A Particular Example - $\hat{Q} = S^3/Z_k$

As a particular example, we consider the three-manifold \hat{Q} to be the lens space S^3/Z_k as in this case the threshold corrections can be computed. In addition, for concreteness, we consider a situation where the first hidden sector gauge group is obtained from a larger group $SU(P + M + 1)$ by Wilson line breaking $SU(P + M + 1) \rightarrow SU(P + 1) \times SU(M) \times U(1)$, while the second hidden sector group is still $SU(Q)$ without breaking. Again we assume one flavor of charged matter Q and \bar{Q} . As long as M is sufficiently smaller than P (such as $< P/2$), we can neglect its contribution to the superpotential and also in moduli stabilization. The calculation of the threshold correction is similar to that in [7]. For the first hidden sector, it is given by $S'_1 = 2(P + 1)T_{\mathcal{O}} + 2MT_{\lambda}$, while for the second one it is $S'_2 = 2QT_{\mathcal{O}}$. T_{λ} and $T_{\mathcal{O}}$ are the relevant torsions:

$$T_{\mathcal{O}} = -\log(k), \quad T_{\lambda} = \log(4 \sin^2(G\pi\lambda/k)), \quad (\text{A9})$$

where $G = P + M + 1$, and λ is an integer specifying the Wilson line. As discussed above, $C_{1,2}$ can be calculated straightforwardly, which are

$$C_1 = M_P^{-3} \langle \text{Vol}(\hat{Q})^{-(1+1/P)} \rangle \Lambda_{\text{cutoff}}^{-1/P} e^{-\frac{S'_1}{2P}} \quad (\text{A10})$$

$$C_2 = M_P^{-3} \langle \text{Vol}(\hat{Q})^{-1} \rangle e^{-\frac{S'_2}{2Q}}. \quad (\text{A11})$$

Here, $\text{Vol}(\hat{Q})$ is the dimensionful volume of the hidden-sector three-manifold \hat{Q} . It is important to remember that C_1, C_2 should be thought of as depending on the vacuum expectation value of $\text{Vol}(\hat{Q})$ ¹⁹ as shown in the above equation. Therefore, this dependence does not invalidate the holomorphicity of the superpotential. One can now compute the P_{eff} from the above equations:

$$\begin{aligned} P_{\text{eff}} &\approx P \left(-\frac{S'_1}{2P} + \frac{S'_2}{2Q} \right) \\ &= -T_{\mathcal{O}} - MT_{\lambda} \\ &= \log(k) - M \log(4 \sin^2(G\pi\lambda/k)) \end{aligned} \quad (\text{A12})$$

¹⁹ i.e. obtained after moduli stabilization

It is obvious that k has to be very large to get a large P_{eff} . For example, $P = 15$, $Q = 18$, $M = 10$, $V_7 = 50$, $\lambda = 80$ and $k = 99$ gives $P_{\text{eff}} = 58$ and $C_2 = 1.5 \times 10^{-4}$. Notice C_2 is much smaller than one. The gravitino for this case is about 0.8TeV. One can also consider other patterns of symmetry breaking, e.g. $SO(2(P+1)) \rightarrow SU(P+1) \times U(1)$. In this case smaller values of k compared to the previous example can give rise to a large P_{eff} , although in general a large k is still needed. Large values of k may seem unrealistic, but that is not clear since the allowed possibilities for compact G_2 manifolds fibred over Lens spaces with large k are not known. In addition, although at present it is not known how to compute the torsion for other three-manifolds, it is possible that a large P_{eff} can be obtained more “naturally” in other examples.

2. More General Possibilities

Other three manifolds might give rise to a large P_{eff} more naturally. Rather than study further explicit examples we give a toy model which illustrates this possibility. The model has parameters which extend the previous example and is given by

$$T_{\mathcal{O}} = -\gamma_0 \log(k), \quad T_{\lambda_1} = \gamma_1 \log(\alpha \sin^2(G\pi\lambda_1/k)), \quad T_{\lambda_2}, T_{\lambda_3}, \dots \quad (\text{A13})$$

where G is an integer and $\gamma_{0,1}$ are determined by the topology of the manifold and are kept as free parameters. α is determined by group theory. In general, there could be other non-trivial torsions $T_{\lambda_2}, T_{\lambda_3}$, etc. depending on how the higher gauge group is broken by the Wilson lines. In order to illustrate the idea, we will restrict to $T_{\mathcal{O}}$ and T_{λ_1} . Let’s again consider the case where the first hidden sector group $SU(P+1)$ arises from the breaking $SU(P+M+1) \rightarrow SU(P+1) \times SU(M) \times U(1)$. Now, it is possible to get both $P_{\text{eff}} \approx 84$ and $C_2 \sim \mathcal{O}(1)$. For example, the set of parameters $\gamma_0 = \gamma_1 = 6.3$, $P = 15$, $Q = 18$, $M = 10$, $V_7 = 50$ and $k = 11$ gives $P_{\text{eff}} = 84.2$ and $C_2 = 5.4$.

The above example was shown just to illustrate the fact that with more general three-manifolds \hat{Q} , it may be possible to obtain a large P_{eff} quite naturally. As another possibility, if \hat{Q} is such that the relevant torsions $T_{\mathcal{O}}, T_{\lambda_1}, T_{\lambda_2}$, etc. in (A13) have a linear dependence on k instead of logarithmic one, it is quite easy to obtain a large P_{eff} naturally. In addition, if there are massive quarks²⁰ which are charged under the hidden sector gauge group, the strong coupling scale will be lowered and both the values of P_{eff} and C_2 will be affected.

To summarize, the values of P_{eff} and C_2 depend crucially on the microscopic details of the hidden sector and can take a wide range of values. Therefore, in our phenomenological analysis, we have simply assumed that P_{eff} and C_2 can take values in the range of phenomenological interest.

APPENDIX B: CONSTRAINTS ON “MICROSCOPIC” PARAMETERS

The vacua in realistic G_2 compactifications are characterized by the “microscopic” parameters $\{N_i, N_i^{sm}, a_i, N, P, Q, C_1, C_2, \delta; i = 1, 2, \dots, N\}$. Phenomenologically relevant quantities however, are sensitive to some parameters directly such as P, Q, C_2 and δ and some of them in combinations such

²⁰ Of course, they should be heavier than the strong coupling scale

as V_7 , $V_{\hat{Q}_{vis}}$ and P_{eff} . As mentioned in section II, these combinations have to satisfy various constraints such as the “supergravity constraint”, the “dS vacuum constraint”, the “unified coupling constraint” and the “unification scale constraint”. As promised, here we discuss the first three in detail. The unification scale constraint has already been discussed in section IV.

We would like to find sets of microscopic parameters which give rise to $\{V_7, V_{\hat{Q}_{vis}}, P_{\text{eff}}\}$ such that the above mentioned constraints can be satisfied. In order to take into account the effects of the parameters $a_i, N_i, N_i^{sm}, i = 1, \dots, N$, we consider the following two extreme cases:

- (a) All a_i are roughly equal, so $a_i \approx \frac{7}{3N}$.
- (b) A few a_i are much larger than the rest, for simplicity we take $a_1 \approx \frac{7}{3}$ and $a_{i \neq 1} \approx 0$.

Since other choices of the above parameters lie in between the two extremes, presumably so would their implications. For later use, it is useful to note that at the meta-stable dS minimum, the moduli are stabilized at [1]:

$$s_i = \frac{a_i}{N_i} \nu, \quad \nu \approx \frac{3P_{\text{eff}}Q}{14\pi(Q-P)} \quad (\text{B1})$$

where $P_{\text{eff}} \equiv P \log\left(\frac{C_1}{C_2}\right)$. Let us first consider the weak supergravity constraint for the phenomenologically interesting dS vacua, which reads:

$$V_7 = \prod_{i=1}^N s_i^{a_i} > 1 \quad (\text{B2})$$

For case (a), we can rewrite the seven dimensional volume as

$$\begin{aligned} V_7 &\approx \left(\frac{7\nu}{3N}\right)^{7/3} \left(\prod_{i=1}^N N_i\right)^{-\frac{7}{3N}} \\ &\approx \left(\frac{7\nu}{3N\bar{N}}\right)^{7/3} \end{aligned} \quad (\text{B3})$$

where \bar{N} is defined to be the geometric mean of the N_i 's. One should keep in mind that \bar{N} can be small even if some of the N_i 's are large, if N is $\mathcal{O}(10)$ or greater. The supergravity constraint then turns out to be:

$$\frac{7\nu}{3N\bar{N}} > 1 \quad \implies \quad \frac{P_{\text{eff}}Q}{2\pi N\bar{N}(Q-P)} > 1 \quad (\text{B4})$$

For the case (b), we have $\left(\frac{a_i}{N_i}\right)^{a_i} \approx 1$ for $i \neq 1$, and so $V_7 = \left(\frac{a_1\nu}{N_1}\right)^{7/3}$. Therefore the constraint turns out to be

$$\frac{a_1}{N_1} \nu > 1 \quad \implies \quad \frac{P_{\text{eff}}Q}{2\pi N_1(Q-P)} > 1 \quad (\text{B5})$$

A typical set of “reasonable” as well as phenomenologically interesting values is $P_{\text{eff}} \sim \mathcal{O}(10 - 100)$, $Q \sim \mathcal{O}(10)$, $Q - P \sim \mathcal{O}(1)$ (but > 3), $\bar{N} \sim \mathcal{O}(1)$ and $N_1 \sim \mathcal{O}(1)$, which easily satisfies the supergravity

constraint in case (b). The supergravity constraint for case (a) is also satisfied for many sets of values of the parameters in the above ranges, although not as easily for case (b). For a general case which lies between (a) and (b), we expect a situation in between the two and the constraint should be satisfied for parameters in the above range. The important point is that this constraint can always be satisfied by an asymmetric distribution of a_i .

Now let us consider the unified coupling constraint. The gauge kinetic function for the visible sector is the volume of the visible three-cycle

$$V_{\hat{Q}_{vis}} = \sum_{i=1}^N N_i^{vis} s_i, \quad (\text{B6})$$

which obeys the following inequality:

$$V_{\hat{Q}_{vis}} > N \left(\prod_{i=1}^N N_i^{sm} \right)^{1/N} \left(\prod_{i=1}^N s_i \right)^{1/N} \quad (\text{B7})$$

For case (a), (B6) can be written as:

$$V_{\hat{Q}_{vis}} > N \left(\prod_{i=1}^N N_i^{sm} \right)^{1/N} \left(\frac{7\nu}{3N\bar{N}} \right) \quad (\text{B8})$$

From Eq.(B4) and assuming $N_i^{sm} > 1$ for all i , we find $V_{\hat{Q}_{vis}} > N$. Since for the MSSM $V_{\hat{Q}_{vis}} = \alpha_M^{-1} \sim \mathcal{O}(25)$, this implies $N \lesssim \mathcal{O}(25)$. Thus, equal values of a_i require the number of moduli to be relatively small. One way out of this is that most of the N_i^{sm} are zero, and only $p \lesssim \mathcal{O}(10)$ of them are nonzero. This however, is non-generic. For case (b), one has $\left(\prod_{i=1}^N s_i \right)^{1/N} \sim 0$. Therefore, inequality (B7) can be easily satisfied. Again, case (b) is more easily satisfied than case (a). For a more general situation lying in between (a) and (b), one expects that the constraint can be satisfied for many sets of values of the microscopic parameters.

Finally, the dS vacuum constraint sets an upper limit on P_{eff} :

$$P_{\text{eff}} < \frac{28(Q - P)}{3(Q - P) - 8} \quad (\text{B9})$$

There is also a rough lower limit on P_{eff} . From the supergravity constraint for cases (a) and (b), one gets:

$$\begin{aligned} P_{\text{eff}} &> \frac{2\pi N \bar{N} (Q - P)}{Q} \quad \text{case (a)} \\ P_{\text{eff}} &> \frac{2\pi N_1 (Q - P)}{Q} \quad \text{case (b)} \end{aligned} \quad (\text{B10})$$

For $Q - P = 3$ ²¹, $Q = \mathcal{O}(10)$, $N = \mathcal{O}(50)$ and $\bar{N} = \mathcal{O}(1)$, $P_{\text{eff}} \gtrsim 50$ for case (a) and $P_{\text{eff}} \gtrsim 1$ for case (b). So we can see the lower limit from the supergravity condition is really mild. However for our solution to be valid, we should always restrict to large P_{eff} , which should be ~ 50 . For fixed $Q - P$ and V_7 , the

²¹ This is preferred phenomenologically as will be seen soon.

lightest gravitino mass is achieved when P_{eff} reaches its maximum which happens when the cosmological constant is tuned to be zero. By increasing $Q - P$, the upper limit of P_{eff} decreases, e.g.,

$$\begin{aligned} Q - P &= 3, & P_{\text{eff}} &< 84, \\ Q - P &= 4, & P_{\text{eff}} &< 28, \\ Q - P &= 5, & P_{\text{eff}} &< 20. \end{aligned} \tag{B11}$$

Therefore solutions with $Q - P = 3$ and $P_{\text{eff}} = 83$ are required in order to have a generic number of moduli and a TeV scale gravitino mass, and to tune the cosmological constant.

APPENDIX C: THRESHOLD CORRECTIONS TO GAUGINO MASSES FROM HEAVY FIELDS

In the following, we show explicitly that the presence of a heavy fields will not change the gaugino masses at low energy. The relevant terms in the Lagrangian at M_{unif} can be schematically written as :

$$\mathcal{L} = \int d^4\theta e^{-K/3} (\tilde{K}_T T^* T + \tilde{K}_{T^c} T^{c*} T^c) + \int d^2\theta \mu_T T^c T + \dots \tag{C1}$$

Here, T and T^c collectively stand for the heavy fields with masses $\mu_T \geq M_{\text{unif}}$. In particular, they could stand for heavy M -theory states as well as heavy states in the low energy GUT multiplet (the Higgs triplets in the $SU(5)$ case with a doublet-triplet mechanism for example). As explained in section II C 3, the heavy KK modes of the GUT multiplet give rise to no corrections to gaugino masses. So, those modes are not included in T, T^c . In the conformal supergravity formalism [32], one introduces a conformal compensator field C and inserts it into the Lagrangian to make it conformally invariant and does calculations. At the end, one “gauge fixes” the conformal compensator field $\langle C \rangle = e^{\langle K \rangle/6}$ and $\langle F^C / C \rangle = m_{3/2}^* - \frac{1}{3} F^m \partial_m K$ to obtain standard $\mathcal{N} = 1, D = 4$ SUGRA²². Here K is the Kähler potential for the moduli and hidden sector meson field ($K = -3 \ln(4\pi^{1/3} V_X) + \bar{\phi}\phi$) in the 4d Einstein frame and \tilde{K}_T is the Kähler metric for the matter field T . So, one writes:

$$\mathcal{L} = \int d^4\theta C C^* e^{-K/3} (\tilde{K}_T T^* T + \tilde{K}_{T^c} T^{c*} T^c) + \int d^2\theta C^3 \mu_T T^c T + \dots \tag{C2}$$

After canonically normalizing the Kähler potential, one gets:

$$\mathcal{L} = \int d^4\theta (\hat{T}^* \hat{T} + T^{c*} \hat{T}^c) + \int d^2\theta \frac{\mu_T}{\sqrt{e^{-2K/3} \tilde{K}_T \tilde{K}_T^c}} \hat{T}^c \hat{T} + \dots \tag{C3}$$

$$\text{where } \hat{T} \equiv (\sqrt{\tilde{K}_T e^{-K/3}}) C T$$

So, the normalized mass of T can be written as $M_T = \frac{\mu_T}{\sqrt{e^{-2K/3} \tilde{K}_T \tilde{K}_T^c}}$. The threshold corrections to the gauge coupling can be written as:

$$\Delta f_a^{T, T^c} = -\frac{1}{16\pi^2} C_a^T \ln\left(\frac{M_T M_{T^*}}{M_{\text{unif}}^2}\right) \tag{C4}$$

²² Our definition of the moduli F terms and the gauge kinetic function f_a^0 are slightly different from that in [18].

Now, using (31), one can write:

$$\begin{aligned}
M_a(M_{\text{unif}}^+) - M_a(M_{\text{unif}}^-) &= g_a^2(M_{\text{unif}}) F^I \partial_I (\Delta f_a^{T, T^c}) \\
\Rightarrow M_a(M_{\text{unif}}^-) &= M_a(M_{\text{unif}}^+) + \frac{g_a^2(M_{\text{unif}})}{16\pi^2} \sum_{T, T^c} C_a^T \left(\frac{F^C}{C} + F^m \partial_m \ln(e^{-2K/3} \tilde{K}_T \tilde{K}_{T^c}) \right) \\
\text{where } M_a(M_{\text{unif}}^+) &= g_a^2(M_{\text{unif}}) \left[\frac{1}{8\pi} F^m \partial_m f_a^0 + \frac{1}{8\pi^2} \sum_{MSSM, T, T^c} C_a^i F^m \partial_m \ln(e^{-K/3} \tilde{K}_i) - \right. \\
&\quad \left. \frac{1}{16\pi^2} (3C_a - \sum_{MSSM, T, T^c} C_a^i) \frac{F^C}{C} \right]
\end{aligned} \tag{C5}$$

So, one finally gets for $M_a(M_{\text{unif}}^-)$:

$$\begin{aligned}
M_a(M_{\text{unif}}^-) &= g_a^2(M_{\text{unif}}) \left[\frac{1}{8\pi} F^m \partial_m f_a^0 + \frac{1}{8\pi^2} \sum_{i=1}^{MSSM} C_a^i F^m \partial_m \ln(e^{-K/3} \tilde{K}_i) - \frac{1}{16\pi^2} (3C_a - \sum_{i=1}^{MSSM} C_a^i) \frac{F^C}{C} \right] \\
&= \frac{g_a^2(\mu)}{8\pi} (F^m \partial_m f_a^0) - \frac{g_a^2(\mu)}{16\pi^2} \left(- (3C_a - \sum_i C_a^i) e^{K/2} W^* + (C_a - \sum_i C_a^i) F^m K_m \right. \\
&\quad \left. + 2 \sum_{i=1}^{MSSM} (C_a^i F^m \partial_m \ln(\tilde{K}_i)) \right) \\
&= M_a^{\text{tree}}(M_{\text{unif}}^-) + M_a^{\text{anom}}(M_{\text{unif}}^-)
\end{aligned} \tag{C6}$$

where i runs only over the MSSM particles. Just above the unification scale, the beta function coefficients corresponded to that of the MSSM and the heavy fields T, T^c . From (C6), we see that the threshold correction to the gaugino masses because of the heavy fields exactly cancels the heavy field contribution to $M_a(M_{\text{unif}}^+)$! *This implies that below the unification scale, we can forget about the heavy fields T, T^c and just take effects of the MSSM particles into account.*

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